

Effect of Slip Condition on Thin Film Flow on a Vertical Cylinder for Lift of Electrically Conducting Power Law Fluid

K. N. Memon¹, Rashid Solangi², M. Afzal Soomro¹

Authors' Affiliation:

¹Department of Mathematics and Statistics, QUEST, Nawabshah, Sindh, Pakistan

²Department of BSRS, Mehran University of Engineering Technology, Jamshoro, Pakistan

Article History:

Submitted: December 17, 2022

Revised: April 23, 2023

Accepted: June 09, 2023

Published online: June 30, 2023

Corresponding author(s):

Names K. N. Memon

Email: knmemon@quest.edu.pk

ABSTRACT

This work is made to explore the problem concerning thin film flow for a lift while considering steady, incompressible, isothermal, electrically conducting Power law fluid (PLF) on a vertically standing cylinder with slip conditions. The non-linear differential equation that is ordinary has been obtained using the continuity and momentum equations by using the Perturbation method focus on free space and slip boundary condition. The exact solution concerning the PLF model is recuperated from these proposed models on substitution of slip parameter $\phi = 0$ and $\epsilon = 0$. A series solution has been obtained, which gives us the velocity profile of fluid, flow rate, and average velocity of fluid flow.

Keywords: Thin film flow; electrically conducting PLF; Slip condition, Perturbation solution.

1 | INTRODUCTION

The flow with slipstream is life-threatening in science, the growth and infeasible in industrialization. The miracle of slip-stream a system or ordered way of doing things has dragged in the deliberation of wide number of scholars. The elementary portion adjacent a robust surface is no long-lasting contemplates the speed of the surface in many confirmable dues. At surface molecule causes a finite divergent speed it skims the surface. The fluid slippage at the strong limits in frequent claims, like, the thick monolayer of hydrophobic octa-ecyltrichlorosilane [1]–[6].

In the ancient background of liquid course through stations, Navier inspected a boundary position of liquid slip-up at solid surface just as $v_z = -\beta S_{rz}|_{r=R}$, where β is “slip” coefficient, S_{rz} is part of stress (extra) tensor and v_z is the speed in the direction of z-axis. In some of the circumstances $\beta = 0$ assures that the slip effect is negligible at the limit.

The phenomena of thin film flow has been thoroughly studied in recent years by academics and researchers using both experimental and theoretical methods [7]–[10]. It is crucial to comprehend the mechanics of these flows since they have numerous and widespread applications in a variety of industries.

These applications cover a variety of fields, including flat panel displays, biomedical engineering, energy efficiency, photovoltaics, energy conversion, and pharmaceuticals, all of which have seen recent expansion [8, 10]. Applications involving the interaction of films with mammalian lung linings, manufactured goods, the flow of surface-active fluids, the manufacture of microchips, condensate motion, painting, the production of microchips, and the dispersion of sauces in food products are particularly noteworthy. All of these applications play an important and fascinating role in these contexts [11], [12].

Presently the power-law of non-Newtonian fluid has been generally focused because of numerical simplicity and nonattainment-to-the-minute applications. The severe development has been accredited in the improvement of diagnostic arrangement and numerical calculations [7], [9], [12]–[15]. This research paper is devoted to the investigation of thin film Non-Newtonian fluid flow with electrically conducting properties of fluid with slip condition. In the flow of thin film, the fluid is in share inadequate strong separator while rest of the surface is permissible to communicate with alternative fluid, for example aviation, here three major situations which set up the groundwork for the preparation of thin films, specifically, gravitational and surface tension and centrifugal forces.

The considered fluid with viscosity is generalized as Newtonian fluids following to PLF Magneto hydrodynamics. We studied the cases of thin film power law Magneto hydrodynamics fluid for boost problem on a cylinder vertically aligned with slip condition. To the superlative of our understanding the perturbation solution is not accounted.

This paper, organized in sections covers the leading equation of power law electrically conducting fluid, formulation and solution of problem in hand, the results and various discussions and we conclude the article with some specified remarks.

2 | GOVERNING EQUATIONS

Setting aside the thermal effects, the incompressible viscous fluid flow is modeled by the equations $\nabla \cdot \mathbf{V} = 0$, and $\rho D\mathbf{V}/Dt = -\nabla p + \rho \mathbf{f} + \nabla \cdot \mathbf{S} + (\mathbf{J} \times \mathbf{B})$. The symbol ρ represents density, considered constant thoroughly, $\mathbf{V}$ be the velocity, P stand for the dynamic pressure, $\mathbf{f}$ represents the body force, $\mathbf{S}$ is the extra stress tensor and $\frac{D}{Dt}$ defines material derivatives. As a result Lorentz force per unit volume be $\mathbf{J} \times \mathbf{B} = [0, 0, -\sigma B_0^2 v_z]$, where σ is the electrical conductivity, $\mathbf{B} = [0, 0, B_0]$ be the uniform magnetic field, here B_0 be the magnetic field, as applied, and $\mathbf{J}$ be the current density $\mathbf{J}$, which is $\mathbf{J} = \sigma[\mathbf{E} + \mathbf{V} \times \mathbf{B}]$, and $\nabla \times \mathbf{B} = \mu_0 \mathbf{J}$

Here $\mathbf{E}$ is the electric field which is not considered in this study and μ_0 be the magnetic permeability. The extra stress tensor defining a PLF model is specified in [7], [8], [16].

3 | FORMULATION OF THE PROBLEM

A reasonable container full with PLF (electrically conducting). A verticle cylinder routes directed up with fixed speed U_0 through container. As the cylinder variations overhead, it plucks up a fluid of a thin film thickness δ . The fluid film starts to drain down and emptying the cylinder appropriate to gravity. The

cylindrical is aligned in xz -coordinate system in such that z -axis along the cylinder in an up direction and r - axis is normal to the cylinder as indicated in Figure. 1 as described in [7], [12]. It is assumed or granted that the cylinder does not conduct electricity. It is also assumed that the magnetic field is given length ways the radial axis. It is assumed that given flow must be steady, uniform laminar and the surface stress properties are immaterial, the main velocity, not equal to zero, portion is in the z -direction. In the light of formulated model the governing equation describing flow is

$$0 = \frac{\eta}{r} \frac{d}{dr} \left(r \left(\frac{dw}{dr} \right)^n \right) - \rho g - \sigma B_0^2 w(r) \quad (1)$$

The equation (1) the differential equation (non-linear), here η represent to the dynamic viscosity, n be the power law index and g is characterized to the acceleration due to gravity. The boundary conditions related to our problem can be written as follows.

$$\frac{dw}{dr} = 0, \text{ where } r \text{ has the form } r = R + \delta, \quad (2)$$

$$w = U_0 - \beta \eta \left(\frac{dw}{dr} \right)^n \text{ at } r = R \quad (3)$$

Here β represent to slip parameter and U_0 be the lift velocity of the cylinder.

Perturbation Solution

Where $w(r, \varepsilon)$ the velocity profile can be quantified as a power series, assumption $\varepsilon = \frac{\sigma B_0^2}{\eta}$ small parameter and given by,

$$w(r, \varepsilon) \approx w_0 + \varepsilon w_1 + \varepsilon^2 w_2 \dots \quad (4)$$

By employing equation (4) into equations (1) – (3) and equating the power of ε yields the system of new problems parallel to their associated conditions given at boundaries. Here we start with the 0th order.

Zeroth order problem

$$\frac{1}{r} \times \frac{d}{dr} \times \left(r \times \left(\frac{dw_0}{dr} \right)^n \right) = \frac{\rho g}{\eta}$$

By using the boundary conditions

$$\frac{dw_0}{dr} = 0, \text{ where } r \text{ has the form } r = R + \delta,$$

$$w_0 = U_0 - \beta \eta \left(\frac{dw_0}{dr} \right)^n \text{ at } r = R$$

First order problem

$$\frac{1}{r} \frac{d}{dr} \left(r n \left(\frac{dw_0}{dr} \right)^{n-1} \frac{dw_1}{dr} \right) - w_0 = 0$$

Complete boundary conditions are,

$$w_1 = -\beta \eta \left(n \left(\frac{dw_0}{dr} \right)^{n-1} \frac{dw_1}{dr} \right) \text{ at } r = R.$$

$$\frac{dw_1}{dr} = 0 \quad \text{at} \quad r = R + \delta$$

Solution of the problem

Velocity Profile

Substitute the solution of velocity obtained by solving zeroth order, first order with related condions in Eq. (4), the considerable calculations is

$$\begin{aligned} w(r) = & U_o + \frac{\beta \rho g}{2R} \left((R+\delta)^2 - R^2 \right) - \left(\frac{\rho g}{2\eta} \right) \sum_{k=0}^{\infty} \left(\frac{1}{n} \right) \frac{(-1)^{(k)} (R+\delta)^{\frac{2}{n}-2k}}{2k - \frac{1}{n} + 1} \left(r^{2k - \frac{1}{n} + 1} - R^{2k - \frac{1}{n} + 1} \right) + \\ & \frac{\beta \eta \varepsilon}{2} \left(U_o + \frac{\beta \rho g}{2R} \left((R+\delta)^2 - R^2 \right) \right) \left((R+\delta)^2 - R^2 \right) + \beta \eta \varepsilon \left(\frac{\rho g}{2\eta} \right) \sum_{k=0}^{\infty} \left(\frac{1}{n} \right) \frac{(-1)^{(k)} (R+\delta)^{\frac{2}{n}-2k}}{2k - \frac{1}{n} + 1} \left\{ \frac{1}{2k - \frac{1}{n} + 3} \right. \\ & \left. \left(R^{2k - \frac{1}{n} + 3} - (R+\delta)^{2k - \frac{1}{n} + 3} \right) - \frac{R^{2k - \frac{1}{n} + 1}}{2} \left(R^2 - (R+\delta)^2 \right) \right\} - \varepsilon \left(\frac{\rho g}{2\eta} \right) \sum_{k=0}^{\infty} \left(\frac{1}{n} \right) \left[\left(-\frac{U_o}{2} + \frac{\beta \rho g}{4R} \left(R^2 - (R+\delta)^2 \right) \right) \right. \\ & \left. \sum_{k=0}^{\infty} \left(\frac{1}{n} \right) \frac{(-1)^{(k)} (R+\delta)^{\frac{2}{n}-2k}}{2k - \frac{1}{n} + 1} \left(r^{2k - \frac{1}{n} + 1} - R^{2k - \frac{1}{n} + 1} \right) - \left(\frac{\rho g}{2\eta} \right) \sum_{k=0}^{\infty} \sum_{l=0}^{\infty} \left(\frac{1}{n} \right) \left(\frac{1-n}{l} \right) \frac{(-1)^{(k+l)} (R+\delta)^{\frac{4}{n}-2k-2l-2}}{2k - \frac{1}{n} + 1} \left\{ \frac{1}{2k - \frac{1}{n} + 3} \right. \right. \\ & \left. \left. \left(r^{2k+2l - \frac{2}{n} + 4} - (R)^{2k+2l - \frac{2}{n} + 4} \right) - \frac{(R+\delta)^{2k - \frac{1}{n} - 3}}{2l - \frac{1}{n} + 1} \left(r^{2l - \frac{1}{n} + 1} - R^{2l - \frac{1}{n} + 1} \right) \right\} - \frac{R^{2k - \frac{1}{n} + 1}}{2} \left(\frac{r^{2l - \frac{1}{n} + 3} - R^{2l - \frac{1}{n} + 3}}{2l - \frac{1}{n} + 3} - \frac{(R+\delta)^2}{\left(2l - \frac{1}{n} + 1 \right)^2} \left(r^{2l - \frac{1}{n} + 1} - R^{2l - \frac{1}{n} + 1} \right) \right) \right] \end{aligned} \quad (5)$$

For instance, let $\varepsilon = 0$, we recuperate the solution of the identical problem with PLF gone out MHD [8] and letting $\beta = 0$ results with no slip situation with MHD and if $\varepsilon = \beta = 0$ MHD with no slip condition attained as [7], [17].

Volume flow Rate

The measurement of volume flow rate is given by the symbol Q , it is specified by

$$Q = \int_0^{2\pi} \int_R^{R+\delta} r \times w(r) dr d\theta = 2 \times \pi \int_R^{R+\delta} r w(r) dr \quad (6)$$

By using equation (5) in (6) then we get

$$\begin{aligned}
Q = \pi & + \frac{\beta \rho g}{2R} \left(R^2 - (R+\delta)^2 \right) \left\{ \sum_{k=0}^{\infty} \left(\frac{1}{n} \right) \frac{(-1)^{(k)} (R+\delta)^{\frac{2}{n}-2k}}{2k - \frac{1}{n} + 1} \left(\frac{1}{2k - \frac{1}{n} + 3} \left((R+\delta)^{\frac{2k-1}{n}+3} - R^{\frac{2k-1}{n}+3} \right) - \right. \right. \\
& \left. \left. \frac{R^{\frac{2k-1}{n}+2}}{2} \left((R+\delta)^2 - R^2 \right) \right) + \varepsilon \left\{ \beta \eta \left[U_o \left((R+\delta)^2 - R^2 \right) - \frac{\beta \rho g}{2R} \left((R+\delta)^2 - R^2 \right) \right] \right\} + \beta \eta \varepsilon \left(\frac{\rho g}{2\eta} \right)^{\frac{1}{n}} \sum_{k=0}^{\infty} \left(\frac{1}{n} \right) \frac{(-1)^{(k)} (R+\delta)^{\frac{2}{n}-2k}}{2k - \frac{1}{n} + 1} \left\{ \frac{1}{2k - \frac{1}{n} + 3} \right. \right. \\
& \left. \left(R^{\frac{2k-1}{n}+3} \left((R+\delta)^2 - R^2 \right) - (R+\delta)^{\frac{2k-1}{n}+3} \left((R+\delta)^2 - R^2 \right) \right) + \frac{R^{\frac{2k-1}{n}+1}}{2} \left((R+\delta)^2 - R^2 \right) \right\} + \frac{\varepsilon}{n} \left(\frac{\rho g}{2\eta} \right)^{\frac{1-n}{n}} \left[-U_o \left((R+\delta)^2 - R^2 \right) \right. \\
& \left. \left. + \frac{\beta \rho g}{2R} \left(R^2 - (R+\delta)^2 \right) \right] \right\} + \left(\frac{\rho g}{2\eta} \right)^{\frac{1}{n}} \left(\frac{1}{2k - \frac{1}{n} + 3} \left((R+\delta)^{\frac{2k-1}{n}+3} - R^{\frac{2k-1}{n}+3} \right) - \frac{R^{\frac{2k-1}{n}+2}}{2} \left((R+\delta)^2 - R^2 \right) \right) \left(\frac{\rho g}{2\eta} \right)^{\frac{1}{n}} \\
& \left. \sum_{k=0}^{\infty} \sum_{l=0}^{\infty} \left(\frac{1}{n} \right) \left(\frac{1-n}{l} \right) \frac{(-1)^{(k+l)} (R+\delta)^{\frac{4}{n}-2k-2l-2}}{2k - \frac{1}{n} + 3} \left\{ \frac{1}{\left(2k - \frac{1}{n} + 3 \right) \left(k+l - \frac{1}{n} + 2 \right)} \left(\frac{(R+\delta)^{2k+2l-\frac{2}{n}+6}}{2k+2l-\frac{2}{n}+6} - \frac{(R)^{2k+2l-\frac{2}{n}+6}}{2} - \frac{(R)^{2k+2l-\frac{2}{n}+4}}{2} \left((R+\delta)^2 - R^2 \right) \right) \right\} \right. \\
& \left. - \frac{(R+\delta)^{\frac{2k-1}{n}+3}}{2l - \frac{1}{n} + 1} \left(\frac{2}{2k - \frac{1}{n} + 3} \left(\frac{(R+\delta)^{\frac{2l-1}{n}+3}}{2l - \frac{1}{n} + 3} - \frac{(R)^{\frac{2l-1}{n}+3}}{2} \left((R+\delta)^2 - R^2 \right) \right) \right) \right\} \left(\frac{(R)^{\frac{2k-1}{n}+1}}{2l - \frac{1}{n} + 3} \right) \\
& \left. \left[\left(\frac{(R+\delta)^{\frac{2l-1}{n}+3}}{2l - \frac{1}{n} + 3} - \frac{(R)^{\frac{2l-1}{n}+3}}{2} \left((R+\delta)^2 - R^2 \right) \right) \frac{R^{\frac{2l-1}{n}+3}}{2} \left((R+\delta)^2 - R^2 \right) \frac{(R+\delta)^2}{2l - \frac{1}{n} + 1} \left(\frac{(R+\delta)^{\frac{2l-1}{n}+3}}{2l - \frac{1}{n} + 3} - \frac{(R)^{\frac{2l-1}{n}+3}}{2} \left((R+\delta)^2 - R^2 \right) \right) \right] \right\} \right]
\end{aligned} \tag{7}$$

Average velocity

The following formula is average velocity

$$\bar{V} = \frac{Q}{\pi((R+\delta)^2 - R^2)} \tag{8}$$

Substituting equation (7) to (8) yields the following complicated solution

$$\begin{aligned}
\bar{V} = \frac{1}{((R+\delta)^2 - R^2)} & + \frac{\beta \rho g}{2R} \left(R^2 - (R+\delta)^2 \right) \left\{ \sum_{k=0}^{\infty} \left(\frac{1}{n} \right) \frac{(-1)^{(k)} (R+\delta)^{\frac{2}{n}-2k}}{2k - \frac{1}{n} + 1} \left(\frac{1}{2k - \frac{1}{n} + 3} \left((R+\delta)^{\frac{2k-1}{n}+3} - R^{\frac{2k-1}{n}+3} \right) - \right. \right. \\
& \left. \left. \frac{R^{\frac{2k-1}{n}+2}}{2} \left((R+\delta)^2 - R^2 \right) \right) + \varepsilon \left\{ \beta \eta \left[U_o \left((R+\delta)^2 - R^2 \right) - \frac{\beta \rho g}{2R} \left((R+\delta)^2 - R^2 \right) \right] \right\} + \beta \eta \varepsilon \left(\frac{\rho g}{2\eta} \right)^{\frac{1}{n}} \sum_{k=0}^{\infty} \left(\frac{1}{n} \right) \frac{(-1)^{(k)} (R+\delta)^{\frac{2}{n}-2k}}{2k - \frac{1}{n} + 1} \left\{ \frac{1}{2k - \frac{1}{n} + 3} \right. \right. \\
& \left. \left(R^{\frac{2k-1}{n}+3} \left((R+\delta)^2 - R^2 \right) - (R+\delta)^{\frac{2k-1}{n}+3} \left((R+\delta)^2 - R^2 \right) \right) + \frac{R^{\frac{2k-1}{n}+1}}{2} \left((R+\delta)^2 - R^2 \right) \right\} + \frac{\varepsilon}{n} \left(\frac{\rho g}{2\eta} \right)^{\frac{1-n}{n}} \left[-U_o \left((R+\delta)^2 - R^2 \right) \right. \\
& \left. \left. + \frac{\beta \rho g}{2R} \left(R^2 - (R+\delta)^2 \right) \right] \right\} + \left(\frac{\rho g}{2\eta} \right)^{\frac{1}{n}} \left(\frac{1}{2k - \frac{1}{n} + 3} \left((R+\delta)^{\frac{2k-1}{n}+3} - R^{\frac{2k-1}{n}+3} \right) - \frac{R^{\frac{2k-1}{n}+2}}{2} \left((R+\delta)^2 - R^2 \right) \right) \left(\frac{\rho g}{2\eta} \right)^{\frac{1}{n}} \\
& \left. \sum_{k=0}^{\infty} \sum_{l=0}^{\infty} \left(\frac{1}{n} \right) \left(\frac{1-n}{l} \right) \frac{(-1)^{(k+l)} (R+\delta)^{\frac{4}{n}-2k-2l-2}}{2k - \frac{1}{n} + 3} \left\{ \frac{1}{\left(2k - \frac{1}{n} + 3 \right) \left(k+l - \frac{1}{n} + 2 \right)} \left(\frac{(R+\delta)^{2k+2l-\frac{2}{n}+6}}{2k+2l-\frac{2}{n}+6} - \frac{(R)^{2k+2l-\frac{2}{n}+6}}{2} - \frac{(R)^{2k+2l-\frac{2}{n}+4}}{2} \left((R+\delta)^2 - R^2 \right) \right) \right\} \right. \\
& \left. - \frac{(R+\delta)^{\frac{2k-1}{n}+3}}{2l - \frac{1}{n} + 1} \left(\frac{2}{2k - \frac{1}{n} + 3} \left(\frac{(R+\delta)^{\frac{2l-1}{n}+3}}{2l - \frac{1}{n} + 3} - \frac{(R)^{\frac{2l-1}{n}+3}}{2} \left((R+\delta)^2 - R^2 \right) \right) \right) \right\} \left(\frac{(R)^{\frac{2k-1}{n}+1}}{2l - \frac{1}{n} + 3} \right) \\
& \left. \left[\left(\frac{(R+\delta)^{\frac{2l-1}{n}+3}}{2l - \frac{1}{n} + 3} - \frac{(R)^{\frac{2l-1}{n}+3}}{2} \left((R+\delta)^2 - R^2 \right) \right) \frac{R^{\frac{2l-1}{n}+3}}{2} \left((R+\delta)^2 - R^2 \right) \frac{(R+\delta)^2}{2l - \frac{1}{n} + 1} \left(\frac{(R+\delta)^{\frac{2l-1}{n}+3}}{2l - \frac{1}{n} + 3} - \frac{(R)^{\frac{2l-1}{n}+3}}{2} \left((R+\delta)^2 - R^2 \right) \right) \right] \right\} \right]
\end{aligned} \tag{9}$$

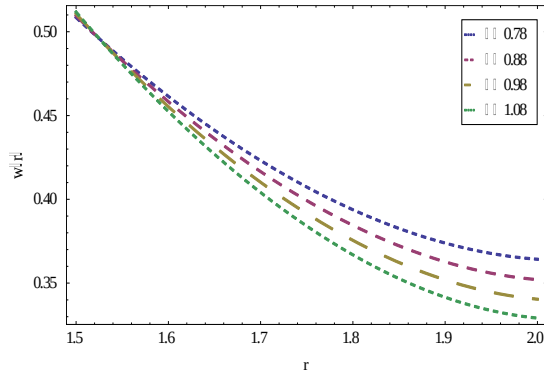


Figure 1: The change caused by ρ on velocity profile for PLF MHD, $\delta=0.5\text{cm}$, $R=1.5\text{cm}$, $\beta=0.002$, $\eta=10\text{poise}$, $\epsilon=0.0021$, $n=1.5$, $U_0=.5$

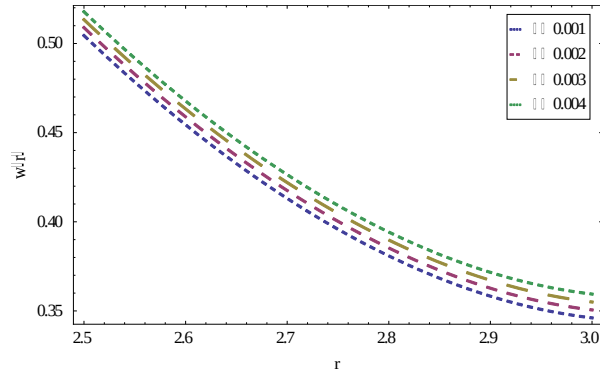


Figure 2: The change because of β on velocity profile for PLF MHD, $\delta=0.5\text{cm}$, $R=2.5\text{cm}$, $\rho=0.88\text{g/cm}^3$, $\eta=10\text{poise}$, $\epsilon=0.004$, $n=1.5$, $U_0=.5$

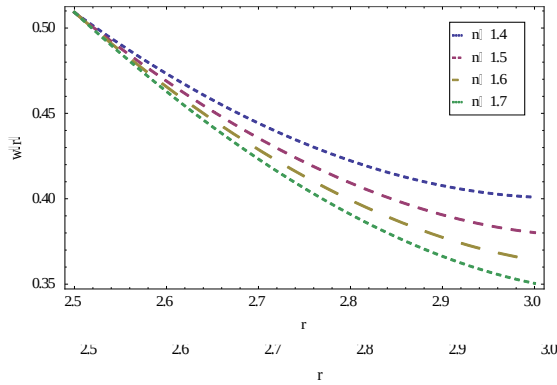


Figure 3: The change caused by ϵ on velocity profile for PLF MHD, $\delta=0.5\text{cm}$, $R=2.5\text{cm}$, $\rho=0.88\text{g/cm}^3$, $\beta=0.002$, $\eta=10\text{poise}$, $n=1.5$, $U_0=.5$

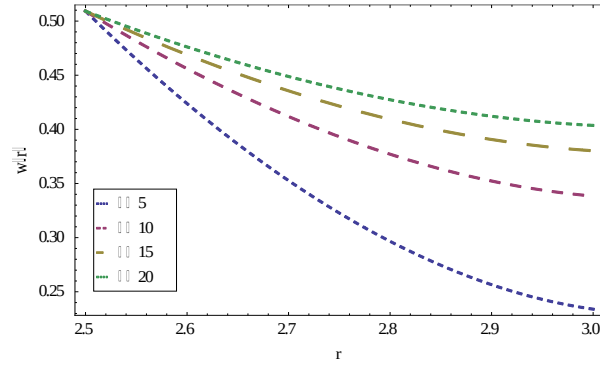


Figure 4: Difference of η on velocity profile for PLF MHD, $\delta=0.5\text{cm}$, $R=2.5\text{cm}$, $\rho=0.88\text{g/cm}^3$, $\beta=0.002$, $\epsilon=0.002$, $n=1.5$, $U_0=.5$

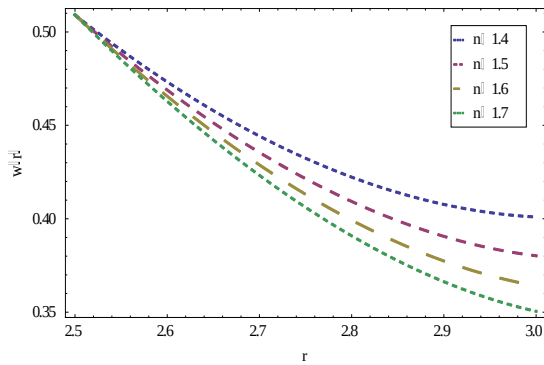


Figure 6: Effect of n on velocity profile for power law MHD fluid, $\delta=0.5\text{cm}$, $R=2.5\text{cm}$, $\rho=0.88\text{g/cm}^3$, $\beta=0.002$, $\eta=15\text{poise}$, $\epsilon=0.002$, $n=1.4$, $U_0=.5$

4 | RESULTS AND DISCUSSION

The lift problem, which deals with the impact of slip circumstances on a vertical cylinder in the setting of thin film flow with an electrically conducting power-law fluid (PLF), was discussed in the parts prior. A set of nonlinear ordinary differential equations are produced as a result of the fluid being described as stable, uniform, isothermal, and incompressible. The fluid's velocity profile and temperature distribution are provided by the perturbation solution we were able to acquire up to the first order. We further

investigated how the velocity profile changes as a function of various parameters. Reliance of flow actions in rate of the Power law index n , slip parameter β , co-efficient of viscosity η , density ρ and magnetic parameter ε are trial actually over and complete with Figures (1) - (5). The nonconformity of velocity (axial) for ε, n, η and β for composed PLF showed, here we point out that, with an enlargement in β, η and ε velocity profile increases, it decreases for the increase of n, ρ .

5 | CONCLUDING REMARKS

In this work we have obtained results in the thin film flow field of a fluid. It is called the PLF MHD with slip condition, on vertically standing cylinder in uplifting problem. The perturbation is the technique by which important differential equations, mostly nonlinear has been determined. The perturbation technique is strong and suitable method for the claimed problem. The flow rate and instantaneous speed and velocity profile, are carried analytically. Here we have point out that with influence magnetic effect and slip condition at the wall of vertical cylinder velocity of the fluid flow for Power-law MHD fluid will be uplift quickly as compare to no slip condition as well as without electrically conducting fluid.

REFERENCES

- [1] D. C. Tretheway and C. D. Meinhardt, "Apparent fluid slip at hydrophobic microchannel walls," *Phys. Fluids*, vol. 14, no. 3, pp. L9–L12, 2002.
- [2] N. S. Shaikh, K. N. Memon, M. S. Sial, and A. M. Siddiqui, "Exact solution on the impact of slip condition for unsteady tank drainage flow of Ellis fluid," 2022.
- [3] M. A. Mahar, K. N. Memon, S. F. Shah, A. A. Amur, and A. M. Siddiqui, "Effect of Slip Condition on Unsteady Tank Drainage Flow of third Order Fluid," 2022.
- [4] S. M. Shah, K. N. Memon, S. F. Shah, A. H. Sheikh, A. A. Ghoto, and A. M. Siddiqui, "Exact solution for ptt fluid on a vertical moving belt for lift with slip condition," *Indian J. Sci. Technol.*, vol. 12, no. 30, p. 22, 2019.
- [5] A. Hussain, K. Ullah, D. Pamucar, I. Haleemzai, and D. Tatić, "Assessment of Solar Panel Using Multiattribute Decision-Making Approach Based on Intuitionistic Fuzzy Aczel Alsina Heronian Mean Operator," *Int. J. Intell. Syst.*, vol. 2023, p. e6268613, May 2023, doi: 10.1155/2023/6268613.
- [6] A. Hussain, H. Wang, K. Ullah, and D. Pamucar, "Novel intuitionistic fuzzy Aczel Alsina Hamy mean operators and their applications in the assessment of construction material," *Complex Intell. Syst.*, Aug. 2023, doi: 10.1007/s40747-023-01116-1.
- [7] K. N. Memon, S. Islam, A. M. Siddiqui, S. A. Khan, N. A. Zafar, and M. Akram, "Lift and drainage of electrically conducting power law fluid on a vertical cylinder," *Appl. Math. Inf. Sci.*, vol. 8, no. 1, p. 45, 2014.
- [8] A. M. Siddiqui, M. Akram, K. N. Memon, S. Islam, and K. Khan, "Withdrawal and drainage of thin film flow on a vertical cylinder," *Sci. Res. Essays*, vol. 41, pp. 3554–3565, 2012.
- [9] M. Farooq, M. T. Rahim, S. Islam, and A. M. Siddiqui, "Withdrawal and drainage of generalized second grade fluid on vertical cylinder with slip conditions," *J. Prime Res. Math.*, vol. 9, no. 1, pp. 51–64, 2013.
- [10] A. Hussain, K. Ullah, T. Senapati, and S. Moslem, "Complex spherical fuzzy Aczel Alsina aggregation operators and their application in assessment of electric cars," *Heliyon*, vol. 9, no. 7, p. e18100, Jul. 2023, doi: 10.1016/j.heliyon.2023.e18100.
- [11] M. K. Alam et al., "Modeling and analysis of high shear viscoelastic Ellis thin liquid film phenomena," *Phys. Scr.*, vol. 96, no. 5, p. 055201, 2021.
- [12] S. A. R. Shah, K. N. Memon, S. F. Shah, A. H. Sheikh, and A. M. Siddiqui, "Delta perturbation method for thin film flow of a third grade fluid on a vertical moving belt," *Stat. Comput. Interdiscip. Res.*, vol. 4, no. 1, pp. 61–73, 2022.
- [13] R. SOLANGI, S. SHAH, A. SIDDIQUI, and K. MEMON, "Effect of slip condition on thin layer flow on an upright cylinder for drainage of electrically conducting power law fluid," *Sindh Univ. Res. J.-SURJ Sci. Ser.*, vol. 48, no. 4, 2016.

- [14] A. Hussain, K. Ullah, J. Ahmad, H. Karamti, D. Pamucar, and H. Wang, "Applications of the Multiattribute Decision-Making for the Development of the Tourism Industry Using Complex Intuitionistic Fuzzy Hamy Mean Operators," *Comput. Intell. Neurosci.*, vol. 2022, 2022.
- [15] A. Hussain, K. Ullah, D. Pamucar, and \Djor\dje Vranješ, "A Multi-Attribute Decision-Making Approach for the Analysis of Vendor Management Using Novel Complex Picture Fuzzy Hamy Mean Operators," *Electronics*, vol. 11, no. 23, p. 3841, 2022.
- [16] K. N. Memon, M. K. Alam, J. Baili, Z. Nawaz, A. H. Shiekh, and H. Ahmad, "Analytical solution of tank drainage flow for electrically conducting newtonian fluid," *Therm. Sci.*, vol. 25, no. Spec. issue 2, pp. 433–439, 2021.
- [17] T. C. Papanastasiou, G. C. Georgin, and A. N. Alexandrou, "CRC Press Boca Raton London Newyork Washington," DC, vol. 6, p. 23, 2000.