

T-Spherical Fuzzy Information and Shweizer-Sklar Operations Based Maclaurin Symmetric Mean Operator and Their Applications

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Article History:

Submitted: July 26, 2023

Revised: November 13, 2023

Accepted: December 07, 2023

Published online: December 30,
2023

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ABSTRACT

T-spherical fuzzy set (TSFS) is the generalization of the fuzzy set (FS) which extracts the information from the real-life scenario with certainty. Aside from the remarkable advantage of being able to account for the connections among the multi-input considerations, such as multi-attributes or multi-experts in the multi-attribute group decision-making (MAGDM), the Maclaurin symmetric mean operator (MSMO) is also the generalization of several different existing operators. Moreover, one important class of T-norms (TN) and T-conorms (TC) is the Schweizer-Sklar TN (SSTN) and TC (SSTC). In this article, the operational laws for TSFS based on SSTN and SSTC are introduced first. Then the introduced operations are used to develop a class of aggregation operators (AOs) to aggregate the information in the form of the T-spherical fuzzy values (TSFVs). The introduced operators in this article are the T-spherical fuzzy Schweizer-Sklar MSMO (TSFSSMSMO) and the T-spherical fuzzy Schweizer-Sklar weighted MSMO (TSFSSWMSMO). Further, the TSFSSMSMO and TSFSSWMSMO are applied to a specific multi-attribute group decision-making (MAGDM) problem to show the significance of the developed operators.

Keywords: Decision-making, information handling, T-spherical fuzzy set, aggregation operators

(MSC2020) Codes: 94D05, 28E10

1 | INTRODUCTION

An important idea to reduce the uncertainty, an FS theory was introduced by [1]. In FS, a degree called membership degree (MD) characterizes all terms that are not well-defined in classical set theory. The value of MD (m) ranges from 0 to 1 to show an object to be the part of a specific scenario. To generalize the theory of FS, an interesting idea of intuitionistic FS (IFS) was introduced by [2]. In IFS, the non-MD (NMD) (n) is an additional degree to show the negative degree of an object to be part of the

specific scenario. The IFS restricts the decision-maker to assign the values to m and n by condition $m + n \leq 1$. This restriction was reduced by [3] when he introduced the idea of the Pythagorean FS (PyFS). In PyFS, the decision-maker are somehow relaxed to assign the values to m and n such that $m^2 + n^2 \leq 1$. To relax more, a model of q-rung orthopair FS (qROFS) was introduced by [4] such that $m^q + n^q \leq 1$. No doubt, the IFS is more reliable than the FS but, it fails when there is a scenario where an object has multiple descriptions such as in scenario of voting (vote for, abstain, vote against, and refusing). To cover the gap, an interesting theory known as picture FDS (PFS) was introduced by [5]. In PFS, an object is described by m and n as well as an abstinence degree (AD) h FROM $[0,1]$ such that $m + h + n \leq 1$. But, the PFS is also restricted because the sum of the degrees exceeds 1 in several cases. To reduce the restrictions, [6] introduced a novel concept of the spherical FS (SFS) and TSFS with conditions $m^2 + h^2 + n^2 \leq 1$ and $m^q + h^q + n^q \leq 1$.

Due to sociological and economic changes, multi-attribute decision-making (MADM) and MAGDM have become widely used in various industrial fields. The aggregation operator (AO) is a crucial tool for tackling MADM and MAGDM problems. Several researchers have investigated the solutions to the MADM and MAGDM problems by introducing new approaches based on FS, IFS, PFS, SFS, and TSFS. Several geometric AOs based on IFS were proposed by [7]. The induced generalized AOs for based on IFS were introduced by [8]. To solve the MAGDM problem, [9] introduced innovative AOs and a ranking mechanism for complex IFS.

In [10], the authors presented several geometric AOs for the framework of the interval-valued IFS to solve MAGDM problems. To solve MADM problems, [11] introduced AOs based on logarithmic operational principles for PyFS. Pythagoras fuzzy power AOs were introduced by [12] and their use in MADM was studied. Pythagorean fuzzy Dombi AOs were suggested by [13] and their use in decision support systems was studied. Several generalized Pythagorean fuzzy Bonferroni mean AOs with their applications in MAGDM were proposed by [14]. In [15], the authors proposed interactionally partitioned Heronian mean AOs on linguistic qROFS. For complex qROFS and related applications, [16] suggested power AOs and VIKOR techniques. For qROFS, [17] suggested several AOs and explained how to use them in MADM. A hybrid decision-making model under qROFS AOs was put forth by [18]. Complex qROFS AOs were proposed by [19] and used in MAGDM. Based on PFS, [20] developed various geometric AOs and their use in MADM. Picture fuzzy Dombi AOs were suggested by [21] and their use in MADM was studied. For picture-hesitant fuzzy collections, [22] presented MADM based on averaging AOs. Frank AOs and an analytical hierarchy procedure based on interval-valued FS and its applications were proposed by [23]. Weighted picture fuzzy AOs were suggested by [24] and applied to MADM. The generalized multimora technique and Dombi prioritized weighted AOs based on TSFS and their applications were proposed by [25]. A policy decision-making based on several averaging AOs for TSFS were proposed by [26]. T-spherical fuzzy Einstein hybrid AOs were suggested by [27] and used in

MADM issues. T-spherical fuzzy Hamacher AOs were suggested by [28] for evaluating the effectiveness of search and rescue robots. T-spherical fuzzy power AOs were suggested by [29] and used in MADM. With application to MADM, [30] suggested Aczel-Alsina aggregation procedures on TSFS.

A few extended AOs have recently been introduced to do some unique duties. There is always some sort of link between various qualities in actual MAGDM situations. Therefore, it is crucial and beneficial to take the relationships between the arguments as a whole into account. Only the MSMO meets this requirement since it uses a variable parameter to account for relationships among any number of arguments. It is better suited to handling MAGDM-specific problems in some real situations where we need to completely account for the interrelationships among many aggregated arguments. Additionally, because Schweizer-Sklar operations can choose parameter values from an infinite set and can be adjusted with varied parameters by decision-makers with varying risk attitudes, such as risk aversion and risk preference, they are more flexible and appropriate for real MAGDM problems. To resolve the MAGDM concerns with TSFVs, it is essential and advantageous to extend MSMO to TSFVs based on Schweizer-Sklar operations. By putting out some new Schweizer-Sklar operating rules and new T-spherical fuzzy MSMO based on these new rules, the goal of this work is to provide a MAGDM technique in the context of the TSFS. The suggested AOs and techniques have the following benefits: (1) they are more adaptable and can take decision-makers risk attitudes into account, and (2) depending on the various choice situations, they can take into account the connections between any number of the aggregated reasons.

This essay's general organization is as follows. In section 2, we primarily introduce some basic concepts such as TSFS, MSMO, SSTT, and SSTC. In section 3, we illustrate operational laws for TSFVS based on SSTN and SSTC and hence are applied to develop TSFSSMSMO and TSFSSWMSMO. Section 4 shows the algorithm to apply the developed AOs to MAGDM problems. Section 5 describes the application of the developed approach to MAGDM problems. Section 6 concludes the study.

2 | PRELIMANRIES

We will introduce some fundamental and practical TSFS, SSTN, SSTC, and MSMO to understand this study.

Definition 1: A TSFS $\mathcal{P}$ on a finite set $U = \{u_1, u_2, u_3, \dots, u_t\}$ is specified as:

$$\mathcal{P} = \{(u_i, (m(u_i), h(u_i), n(u_i)))\}$$

Where $m(u_i)$, $h(u_i)$ and $n(u_i)$ denote MD, AD, and NMD such that $0 \leq m^q(u_i) + h^q(u_i) + n^q(u_i) \leq$

1. The refusal degree (RD) is defined as $\ell(u_i) = \sqrt[q]{1 - (m^q(u_i) + h^q(u_i) + n^q(u_i))}$.

Definition 2: SSTN and SSTCN for $(x, y) \in [0, 1]^2$ are defined as

$$T_{SS, \kappa}(x, y) = (x^\kappa + y^\kappa - 1)^{1/\kappa}$$

$$T_{SS}^*, \kappa(x, y) = 1 - ((1-x)^\kappa + (1-y)^\kappa - 1)^{\frac{1}{\kappa}}$$

Where $\kappa < 0$.

Definition 3: Consider $\lambda = (m, h, n)$ be the TSFV. Then the score value of the TSFV can be defined as the following

$$scr(\lambda) = m^q - h^q - n^q$$

Definition 4: Consider $\tau_e (e = 1, 2, \dots, \eta)$ is a collection of real numbers that are not negative, and $e = 1, 2, \dots, \eta$. Then the MSMO is defined as

$$MSMO^{(\xi)}(\tau_1, \tau_2, \dots, \tau_\eta) = \left(\frac{\sum_{1 \leq e_1 < \dots < e_\xi \leq \eta} \prod_{j=1}^{\xi} \tau_{e_j}}{C_\eta^\xi} \right)^{1/\xi}$$

Where $C_\eta^\xi = \frac{\eta!}{\xi!(\eta-\xi)!}$ is the binomial coefficient, $(e_1, e_2, \dots, e_\xi)$ traverses all the k-tuple combination of $(1, 2, \dots, m)$, where $1 \leq \xi \leq \eta$. For instance, if $\eta = 4$ and $\xi = 3$, then $\sum_{1 \leq e_1 < \dots < e_\xi \leq \eta} \prod_{j=1}^{\xi} \tau_{e_j} = \tau_1 \tau_2 \tau_3 + \tau_1 \tau_2 \tau_4 + \tau_1 \tau_3 \tau_4 + \tau_2 \tau_3 \tau_4$

The MSMO possesses the following characteristics

$$\begin{aligned} MSMO^{(\xi)}(0, 0, \dots, 0) &= 0, MSMO^{(\xi)}(\tau, \tau, \dots, \tau) = \tau; \\ MSMO^{(\xi)}(\tau_1, \tau_2, \dots, \tau_\eta) &\leq MSMO^{(\xi)}(b_1, b_2, \dots, b_\eta), \text{ if } \tau_e \leq b_e \text{ for all } e; \\ \min_e \{\tau_e\} &\leq MSMO^{(\xi)}(\tau_1, \tau_2, \dots, \tau_\eta) \leq \max_e \{\tau_e\}. \end{aligned}$$

3 | DEVELOPMENT OF TSFSSMSMO AND TSFSSWMSM

This section introduces two new operators based on the SSTN and SSTC operational rules of TSFVs: TSFSSMSMO and TSFSSWMSMO. The properties of these new operators are then proven, and some special cases are discussed. To develop new AOs first we define some operational laws for TSFVs based on SSTN and SSTC.

Definition 5: Consider $\lambda = (m, h, n)$, $\lambda_1 = (m_1, h_1, n_1)$ and $\lambda_2 = (m_2, h_2, n_2)$ be three TSFVs. Then the SSTN and SSTC-based operations are defined as follows.

$$\begin{aligned} \lambda_1 \oplus_{ss} \lambda_2 &= \left(\begin{array}{c} \left(1 - ((1 - m_1^q)^\kappa + (1 - m_1^q)^\kappa)^{\frac{1}{\kappa}} \right)^{\frac{1}{q}}, \\ (h_1^\kappa + h_2^\kappa - 1)^{\frac{1}{\kappa}}, \\ (n_1^\kappa + n_2^\kappa - 1)^{\frac{1}{\kappa}} \end{array} \right) \\ \lambda_1 \otimes_{ss} \lambda_2 &= \left(\begin{array}{c} (m_1^\kappa + m_2^\kappa - 1)^{\frac{1}{\kappa}}, \\ \left(1 - ((1 - h_1^q)^\kappa + (1 - h_1^q)^\kappa)^{\frac{1}{\kappa}} \right)^{\frac{1}{q}}, \\ \left(1 - ((1 - n_1^q)^\kappa + (1 - n_1^q)^\kappa)^{\frac{1}{\kappa}} \right)^{\frac{1}{q}} \end{array} \right) \\ \eta \lambda_1 &= \left(\begin{array}{c} 1 - (\eta(1 - m_1)^\kappa - (\eta - 1))^{1/\kappa} \\ (\eta h_1^\kappa - (\eta - 1))^{\frac{1}{\kappa}} \\ (\eta n_1^\kappa - (\eta - 1))^{1/\kappa} \end{array} \right) \end{aligned}$$

$$\lambda_1^\eta = \begin{pmatrix} (\eta m_1^\kappa - (\eta - 1))^{\frac{1}{\kappa}}, \\ 1 - (\eta(1 - h_1)^\kappa - (\eta - 1))^{\frac{1}{\kappa}} \\ 1 - (\eta(1 - h_1)^\kappa - (\eta - 1))^{1/\kappa} \end{pmatrix}$$

Theorem 1. Let $\lambda_1 = (m_1, h_1, n_1)$ and $\lambda_2 = (m_2, h_2, n_2)$ be any two TSFVs, and $\kappa < 0$. Then

$$\begin{aligned} \lambda_1 \oplus_{ss} \lambda_2 &= \lambda_2 \oplus \lambda_2 \oplus_{ss} \lambda_1, \\ \lambda_1 \otimes_{ss} \lambda_2 &= \lambda_2 \oplus \lambda_2 \otimes_{ss} \lambda_1 \\ \eta(\lambda_1 \oplus_{ss} \lambda_2) &= \eta \lambda_1 \oplus_{ss} \eta \lambda_2, \eta \geq 0 \\ \eta_1 \lambda_1 \oplus_{ss} \eta_2 \lambda_1 &= (\eta_1 + \eta_2) \lambda_1, \eta_1, \eta_2 \geq 0 \\ \lambda_1^{\eta_1} \otimes \lambda_1^{\eta_2} &= \lambda_1^{\eta_1 + \eta_2}, \eta_1, \eta_2 \geq 0, \\ \lambda_1^\eta \otimes_{ss} \lambda_2^\eta &= (\lambda_1 \otimes_{ss} \lambda_2)^\eta, \eta \geq 0. \end{aligned}$$

Now, we develop TSFSSMSMO based on developed operational laws for TSFVs as follows.

Definition 6: Consider $\lambda_e (e = 1, 2, \dots, \eta)$ is a collection of TSFVs, $\xi = 1, 2, \dots, \eta$. Then TSFSSMSMO is defined as

$$TSFSSMSMO^{(\xi, \kappa)}(\lambda_1, \lambda_2, \dots, \lambda_\eta) = \left(\frac{\oplus_{1 \leq e_1 < \dots < e_\xi \leq \eta} ss \otimes_{j=1}^\xi ss \lambda_{e_j}}{C_\eta^\xi} \right)^{\frac{1}{\xi}}$$

Some interesting properties of TSFSSMSMO are stated as follows.

Theorem 2. Suppose $\lambda_e = (m_e, h_e, n_e)$ is a set of TSFVs and $\kappa < 0, \xi = 1, 2, \dots, \eta$. Then the result obtained from the TSFSSWMSMO is again a TSFV and

$$\begin{aligned} & TSFSSMSMO^{(\xi, \kappa)}(\lambda_1, \lambda_2, \dots, \lambda_\eta) \\ &= \begin{pmatrix} \left(\left(\left(\frac{1}{\xi} \left(1 - \left(\frac{1}{C_\eta^\xi} \left(\sum_{1 \leq e_1 < \dots < e_\xi \leq \eta} (m_{e_j})^\kappa \right) - (\xi - 1) \right) \right)^{\frac{1}{\kappa}} \right)^\kappa \right)^{\frac{1}{\kappa}} \right)^\kappa, \\ 1 - \left(\frac{1}{\xi} \left(1 - \left(\frac{1}{C_\eta^\xi} \sum_{1 \leq e_1 < \dots < e_\xi \leq \eta} \left(1 - \left(\left(\sum_{j=1}^\xi (h_{e_j})^\kappa - (\xi - 1) \right)^{\frac{1}{\kappa}} \right)^\kappa \right)^{\frac{1}{\kappa}} \right)^\kappa - \left(\frac{1}{\xi} - 1 \right) \right)^{\frac{1}{\kappa}}, \\ 1 - \left(\frac{1}{\xi} \left(1 - \left(\frac{1}{C_\eta^\xi} \sum_{1 \leq e_1 < \dots < e_\xi \leq \eta} \left(1 - \left(\left(\sum_{j=1}^\xi (n_{e_j})^\kappa - (\xi - 1) \right)^{\frac{1}{\kappa}} \right)^\kappa \right)^{\frac{1}{\kappa}} \right)^\kappa - \left(\frac{1}{\xi} - 1 \right) \right)^{1/\kappa} \right) \end{pmatrix} \end{aligned}$$

Proof: Proof is skipped.

Theorem 3: (Idempotency) Suppose $\lambda_e = (m_e, h_e, n_e) (e = 1, 2, \dots, \eta)$ is a collection of the TSFVs, if $\lambda_e = \lambda = (m, h, n)$. Then

$$TSFSSMSMO^{(\xi, \kappa)}(\lambda_1, \lambda_2, \dots, \lambda_\eta) = \lambda = (m, h, n)$$

Proof: Proof is skipped.

Theorem 4: (Monotonicity) Suppose $\lambda_e = (m_e, h_e, n_e)$ and $\lambda'_e = (m'_e, h'_e, n'_e)$ are two sets of TSFVs, if $m_e \geq m'_e, h_e \leq h'_e$ and $n_e \leq n'_e$ for all $e = 1, 2, \dots, \eta$. Then

$$TSFSSMSMO^{(\xi, \kappa)}(\lambda'_1, \lambda'_2, \dots, \lambda'_\eta) \geq TSFSSMSMO^{(\xi, \kappa)}(\lambda_1, \lambda_2, \dots, \lambda_\eta)$$

Proof: Proof is skipped.

Theorem 5: (Boundedness) Suppose $\lambda_e = (m_e, h_e, n_e)$ is a set of TSFVs, if $m^- = \min_{1 \leq e \leq \eta} \{m_e\}, m^+ = \max_{1 \leq e \leq \eta} \{m_e\}, h^- = \min_{1 \leq e \leq \eta} \{h_e\}, h^+ = \max_{1 \leq e \leq \eta} \{h_e\}, n^- = \min_{1 \leq e \leq \eta} \{n_e\}, n^+ = \max_{1 \leq e \leq \eta} \{n_e\}$, let $\lambda^- = (m^-, h^+, n^+)$ and

$$\lambda^+ = (m^+, h^-, n^-), \text{ then}$$

$$\lambda^- \leq TSFSSMSMO^{(\xi, \kappa)}(\lambda_1, \lambda_2, \dots, \lambda_\eta) \leq \lambda^+$$

Proof: Proof is skipped.

Even though the TSFSSMSMO can take into account the relationships between many aggregated arguments, it does not take into account how important each aggregated argument is on its own. To address this flaw, the TSFSSWMSMO is defined in the following.

Definition 7: Suppose λ_e is a collection of TSFVs and $\xi = 1, 2, \dots, \eta$. Then TSFSSWMSMO is defined as follows

$$TSFSSWMSMO^{(\xi, \kappa)}(\lambda_1, \lambda_2, \dots, \lambda_\eta) = \left(\frac{1 \leq e_1 < \dots \oplus_{ss} < e_\xi \leq \eta \text{ }_{j=1}^{\xi} \otimes SS(\ell_{e_j} \lambda_{e_j})}{C_\eta^\xi} \right)^{\frac{1}{\xi}}$$

Some basic properties of TSFSSWMSMO are stated as follows.

Theorem 6. Suppose $\lambda_e = (\mu_e, h_e, n_e)$ is a collection of TSFVs. Then, the aggregated result from TSFSSWMSMO is still a TSFV and

$$TSFSSWMSMO^{(\xi, \kappa)}(\lambda_1, \lambda_2, \dots, \lambda_\eta)$$

$$= \left(\left(\left(\left(\left(\left(\frac{1}{\xi} \left(1 - \left(\frac{1}{C_\eta^\xi} \left(\sum_{1 \leq e_1 < \dots < e_\xi \leq \eta} \left(1 - \left(\sum_{e=1}^{\xi} \left(1 - \left(\frac{\ell_{e_j} (1 - \mu_{e_j})^\kappa}{(\ell_{e_j} - 1)} \right)^{\frac{1}{\kappa}} \right) - (\xi - 1) \right)^{\frac{1}{\kappa}} \right) \right)^{\frac{1}{\kappa}} \right) \right)^{\frac{1}{\kappa}} \right) \right)^{\frac{1}{\kappa}} \right) - \left(\frac{1}{\xi} - 1 \right) \right)^{\frac{1}{\kappa}} \right)$$

$$= \left(1 - \left(\frac{1}{\xi} \left(1 - \left(\frac{1}{C_\eta^\xi} \left(\sum_{1 \leq e_1 < \dots < e_\xi \leq \eta} \left(1 - \left(\sum_{j=1}^{\xi} \left(1 - \left(\frac{\ell_{e_j} h_{e_j}^\kappa}{(\ell_{e_j} - 1)} \right)^{\frac{1}{\kappa}} \right) - (\xi - 1) \right)^{\frac{1}{\kappa}} \right) \right)^{\frac{1}{\kappa}} \right) \right)^{\frac{1}{\kappa}} \right) - \left(\frac{1}{\xi} - 1 \right) \right)^{\frac{1}{\kappa}} \right)$$

$$= \left(1 - \left(\frac{1}{\xi} \left(1 - \left(\frac{1}{C_\eta^\xi} \left(\sum_{1 \leq e_1 < \dots < e_\xi \leq \eta} \left(1 - \left(\sum_{j=1}^{\xi} \left(1 - \left(\frac{\ell_{e_j} n_{e_j}^\kappa}{(\ell_{e_j} - 1)} \right)^{\frac{1}{\kappa}} \right) - (\xi - 1) \right)^{\frac{1}{\kappa}} \right) \right)^{\frac{1}{\kappa}} \right) \right)^{\frac{1}{\kappa}} \right) - \left(\frac{1}{\xi} - 1 \right) \right)^{\frac{1}{\kappa}} \right)$$

Theorem 7: (Monotonicity) Consider $\lambda_e = (\mu_e, h_{e_j}, n_{e_j})$ and $\lambda'_e = (\mu'_e, h'_{e_j}, n'_{e_j})$ are two sets of TSFVs, if $\mu_j \geq \mu'_j, h_{e_j} \leq h'_{e_j}$ and $n_j \leq n'_j$ for all $e = 1, 2, \dots, \eta$, then

$$TSFSSWMSMO^{(\xi, \kappa)}(\lambda_1, \lambda_2, \dots, \lambda_\eta) \geq TSFSSWMSMO^{(\xi, \kappa)}(\lambda'_1, \lambda'_2, \dots, \lambda'_\eta)$$

Theorem 8: (Boundedness) Suppose $\lambda_e = (\mu_e, h_{e_j}, n_{e_j})$ is a set of TSFVs and $\lambda^- = (m_{men}^-, h_{men}^+, n_{men}^+)$, $\lambda^+ = (\mu_{max}^+, h_{max}^-, n_{max}^-)$. Then

$$\lambda^- \leq TSFSSWMSMO(\lambda_1, \lambda_2, \dots, \lambda_\eta) \leq \lambda^+$$

4 | DECISION ALGORITHM

The developed approach can be applied to solve the problem of MAGDM. Let $\mathcal{B} = \{\mathcal{B}_1, \mathcal{B}_2, \dots, \mathcal{B}_m\}$ be the set of alternatives and $\wp = \{\wp_1, \wp_2, \dots, \wp_\eta\}$ is the attribute's list having weights $\ell = (\ell_1, \ell_2, \dots, \ell_\eta)^T$ with $\ell_e \geq 0$, $e = 1, 2, \dots, \eta$, and $\sum_{e=1}^{\eta} \ell_e = 1$. Consider, $\{\mathcal{G}_1, \mathcal{G}_2, \dots, \mathcal{G}_t\}$ be experts having weights $\omega = (\omega_1, \omega_2, \dots, \omega_t)$. Consider, that the obtained decision matrix from each expert is denoted by $\lambda^s = [r_{ej}^s]_{m \times \eta}$, where $r_{ej}^s = (m_{ej}, h_{ej}, n_{ej})$ is the evaluation information of alternative $\mathcal{B}_e$ concerning the criteria $\wp_j$ assigned by expert $\mathcal{G}_s$ in the form of the TSFVs. We have to rank the alternatives based on the provided list of attributes based on the developed AOs. We will provide a detailed decision-making process based on the suggested TSFSSWMSM operator, which is illustrated as follows.

Step 1. Normalize the information used to make decisions.

Benefit attributes and cost attributes are the two main categories of characteristics. By normalizing the decision matrix, $\lambda^s = [r_{ej}^s]_{m \times \eta}$, we can eliminate the effects of various attribute types before aggregating the evaluated attribute values. The normalized decision matrix is given as $\lambda^s = [r_{ej}^s]_{m \times \eta}$ after converting the cost attribute values to benefits ones, $(e = 1, 2, \dots, m; j = 1, 2, \dots, \eta)$,

$$\text{Where, } \lambda_{ej}^s = \begin{cases} (m_{ej}, h_{ej}, n_{ej}) & \text{for benefit attributes} \\ (n_{ej}, h_{ej}, m_{ej}) & \text{for cost attributes} \end{cases}$$

Step 2. Using TSFSSWMSMO, we can obtain the aggregate of the information obtained from the experts as follows.

$$r_{ej} = TSFSSWMSM^{(\xi, \kappa)}(r_{ej}^1, r_{ej}^2, \dots, r_{ej}^t)$$

Step 3. To obtain the aggregated information from step 2, we again aggregate the information for each attribute using TSFSSWMSMO as follows.

$$z_e = TSFSSWMSM^{(\xi, \kappa)}(r_{e1}, r_{e2}, \dots, r_{e\eta})$$

Step 4. To obtain the ranking of the alternatives, we find the score values of the alternatives using information obtained in step 3.

Step 5. Prioritize the options.

5 | APPLICATIONS IN MAGDM PROBLEM

In this section, we illustrate how to choose the best investment from five potential companies $\{B1, B2, B3, B4\}$. Three experts g_s ($s = 1, 2, 3$) with weights $\omega = (0.35, 0.40, 0.25)$ evaluate the alternatives based on the attributes having weights $\ell = (0.2, 0.1, 0.3, 0.4)T$. The attributes based on which the alternatives are evaluated are given below.

1. Involved risk ($\wp_1$),
2. The factor of growth ($\wp_2$)
3. Impact of social and political ($\wp_3$)
4. Factor of environment ($\wp_4$).

Each expert evaluates the alternatives based on the given attributes and tabulated in Tables 1-3 as follows.

Table 1: Decision matrix obtained based on provided attributes from the first expert

	$\wp_1$			$\wp_2$			$\wp_3$			$\wp_4$		
B_1	0.35	0.56	0.77	0.33	0.52	0.68	0.23	0.32	0.48	0.53	0.55	0.22
B_2	0.66	0.78	0.34	0.50	0.78	0.55	0.56	0.33	0.58	0.66	0.43	0.45
B_3	0.57	0.69	0.78	0.75	0.56	0.44	0.53	0.56	0.45	0.33	0.65	0.75
B_4	0.58	0.31	0.22	0.45	0.22	0.35	0.00	0.29	0.31	0.44	0.28	0.31

Table 2: Decision matrix obtained based on provided attributes from the second expert

	$\wp_1$			$\wp_2$			$\wp_3$			$\wp_4$		
B_1	0.26	0.19	0.35	0.54	0.23	0.18	0.35	0.21	0.15	0.33	0.44	0.65
B_2	0.42	0.34	0.19	0.17	0.25	0.32	0.16	0.31	0.26	0.32	0.32	0.76
B_3	0.25	0.35	0.18	0.23	0.12	0.51	0.28	0.11	0.16	0.29	0.43	0.25
B_4	0.55	0.22	0.33	0.21	0.32	0.28	0.41	0.31	0.22	0.32	0.11	0.26

Table 3: Decision matrix obtained based on provided attributes from the first expert

	$\wp_1$			$\wp_2$			$\wp_3$			$\wp_4$		
B_1	0.53	0.21	0.18	0.25	0.31	0.17	0.22	0.31	0.15	0.43	0.65	0.24
B_2	0.28	0.19	0.33	0.33	0.16	0.24	0.31	0.11	0.22	0.34	0.82	0.56
B_3	0.13	0.45	0.28	0.36	0.22	0.11	0.19	0.31	0.33	0.40	0.36	0.87
B_4	0.25	0.15	0.36	0.16	0.26	0.36	0.41	0.22	0.36	0.22	0.25	0.33

The following process is illustrated for the suggested method:

Step 1: Normalizing the information used for decision-making λ^s . Here, due to the absence of a cost attribute, we will not normalize.

Step 2: Using TSFSSWMSMO, we can obtain the group evaluation values r_{ej} ($e = 1, 2, 3, 4; j = 1, 2, 3, 4$) (consider $\xi = 2$ and $\kappa = -6$).

$$\begin{aligned}
 r_{11} &= (0.5282, 0.9182, 0.9346), r_{12} = (0.5273, 0.8219, 0.9581), r_{13} = (0.5021, 0.8677, 0.9670), \\
 r_{14} &= (0.5366, 0.2668, 0.8563), r_{21} = (0.5772, 0.9190, 0.9101), r_{22} = (0.5184, 0.9941, 0.7831), \\
 r_{23} &= (0.5302, 0.7974, 0.8500), r_{24} = (0.5761, 0.6095, 0.2017), r_{31} = (0.5263, 0.5539, 0.9433), \\
 r_{32} &= (0.6483, 0.8588, 0.8205), r_{33} = (0.5188, 0.7974, 0.9941), r_{34} = (0.5112, 0.5374, 0.7816), \\
 r_{41} &= (0.5660, 0.9821, 0.8491), r_{42} = (0.5107, 0.8503, 0.7200), r_{43} = (0.5143, 0.8449, 0.8530), \\
 r_{44} &= (0.5134, 0.7914, 0.7651)
 \end{aligned}$$

Step 3: Using TSFSSWMSMO, we obtain the total aggregated value for each alternative as follows.

$$\begin{aligned}
 z_1 &= (0.6170, 0.6280, 0.7460), z_2 = (0.6510, 0.7300, 0.7010), z_3 = (0.5780, 0.7120, 0.7440), \\
 z_4 &= (0.6060, 0.2340, 0.1490)
 \end{aligned}$$

Step 4: We evaluate the scores of each alternative as follows

$$S(z_1) = 0.6368, S(z_2) = 0.7099, S(z_3) = 0.6496, S(z_4) = 0.2227$$

Step 5: Rank the alternatives as follows

$$B_2 > B_3 > B_1 > B_4$$

The involvement of the parameter κ covers the SSTN and SSTC into flexible operational laws. However, a change in the parameter may affect the results. Hence, the obtained results are tabulated in Table 4 as follows.

Table 4: Decision matrix obtained based on provided attributes from the first expert

κ	Scores	Preferences
$\kappa = -6$	$S(z_1) = 0.6368, S(z_2) = 0.7099, S(z_3) = 0.6496, S(z_4) = 0.2227$	$\mathcal{B}_2 > \mathcal{B}_3 > \mathcal{B}_1 > \mathcal{B}_4$
$\kappa = -10$	$S(z_1) = 0.2098, S(z_2) = 0.3708, S(z_3) = 0.1986, S(z_4) = 0.1663$	$\mathcal{B}_2 > \mathcal{B}_1 > \mathcal{B}_3 > \mathcal{B}_4$
$\kappa = -20$	$S(z_1) = 0.2159, S(z_2) = 0.3735, S(z_3) = 0.0967, S(z_4) = 0.2148$	$\mathcal{B}_2 > \mathcal{B}_1 > \mathcal{B}_4 > \mathcal{B}_3$
$\kappa = -30$	$S(z_1) = 0.2416, S(z_2) = 0.3924, S(z_3) = 0.1166, S(z_4) = 0.2415$	$\mathcal{B}_2 > \mathcal{B}_1 > \mathcal{B}_4 > \mathcal{B}_3$
$\kappa = -40$	$S(z_1) = 0.2541, S(z_2) = 0.4009, S(z_3) = 0.1311, S(z_4) = 0.2541$	$\mathcal{B}_2 > \mathcal{B}_1 > \mathcal{B}_4 > \mathcal{B}_3$
$\kappa = -50$	$S(z_1) = 0.2613, S(z_2) = 0.4056, S(z_3) = 0.1402, S(z_4) = 0.2613$	$\mathcal{B}_2 > \mathcal{B}_1 > \mathcal{B}_4 > \mathcal{B}_3$

Now, Figure 1 represents the graphical representation of Table 4, which also indicates that $\mathcal{B}_2$ is the best alternative.

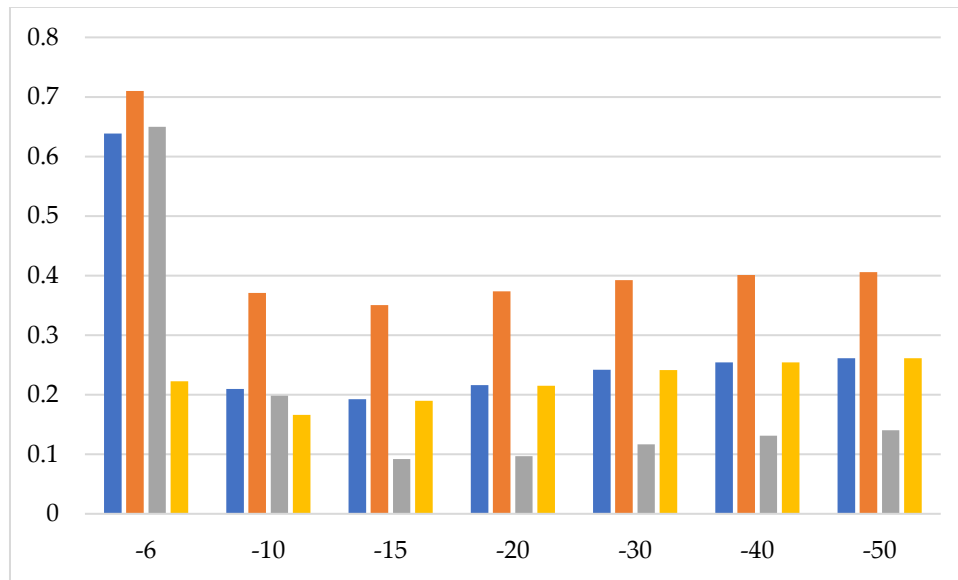


Figure 1: Ranking results of different parameter values of κ

6 | CONCLUSION

By using a variable and infinite parameter, Schweizer-Sklar operations are more adaptable than algebraic operations, and the MSMO is a crucial tool for addressing information fusion issues. The MSMO based on Schweizer-Sklar operations in a T-spherical fuzzy environment hasn't been studied, though. We have created two new Schweizer-Sklar MSMOs for TSFVSs to combine their benefits: the TSFSSMSMO and the TSFSSWMSMO. Additionally, we have solved a MAGDM problem using TSFSSWMSMO. The newly proposed approach is more versatile due to the involvement of the parameter. It is also capable of dealing with independent integrated arguments in addition to taking into consideration the links between various integrated arguments. Additionally, it takes the parameter gamma into account when determining the decision-maker's risk preferences. Therefore, the suggested approach enhances the study of fuzzy decision-making techniques and is better suited to tackle actual and difficult MAGDM situations. We can also generalize this study to the frameworks defined in [31], [32] and [33].

Acknowledgment: We are thankful to the editors and anonymous reviewers for their constructive suggestions.

Author Contributions: Conceptualization, A.H.; methodology, S.Y., and A.H.; software, M.N.A. and M.H.; validation, A.H. and S.Y.; investigation, M.H., M.N.A. and A.H.; data curation, S.Y. and A.H.; writing—original draft preparation, M.N.A.; writing—review and editing, M.H. All authors have read and agreed to the published version of the manuscript.

Declaration of Conflicting Interest: The author(s) declared no potential conflicts of interest with respect to the research, authorship, and/or publication of this article.

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Funding: The author(s) received no financial support for the research, authorship, and/or publication of this article.

Data Availability Statement: Data that supports the findings of this study are available on request from the corresponding author.

Plagiarism Statement: This article was scanned by the plagiarism program. No plagiarism was detected.

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