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Solutions of Mono and Multi-objective Assignment Problems without Modification Matrix

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ABSTRACT

This paper presents a new solution technique for assignment problems. In the first phase, the proposed technique is used for the solution of mono-objective optimization problems, while in the second phase, the proposed technique is extended for the solution of multi-objective optimization problems. In the first phase, a distance matrix is defined and then reduced to the form which contains at least one zero in each row or one zero in each column. Moreover, the sum of each column's entries must be less than 2 to use the Hungarian algorithm. The proposed technique differentiates itself from the existing techniques in the literature in the sense that the case $N \neq n$ will not happen, and consequently, we do not define a modification matrix. Therefore, the proposed methodology reduces the solution procedure to a high extent. The proposed methodology is equally applicable to the solutions of multi-objective optimization problems as well. In such a case, we minimize each objective of the multi-objective problem by the proposed methodology and then use available multi-objective optimization algorithms for further solution for non-dominated solutions. The proposed approach is validated by numerical examples in both phases.

Keywords: assignment problem, optimization, distance matrix, traveling salesman problem, multi-objective optimization.

(MSC2020) Codes: 90C27, 90C29, 90C30

1 | INTRODUCTION

The traveling salesman problem (TSP) was first introduced in 1930, which is one of the vital problems in the sub- branch of graph theory known as combinatorial optimization. The easiest manner in which the problem is defined in combination with its tarnished exertion has encouraged many experts to determine the aggregate or trade-off solution procedure. In combinatorial optimization, the vertices of a graph represent cities that are directly connected through edges, where

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the weight of each edge represents the distance between two cities, for instance, a minimum spanning tree of a weighted planar graph. More specifically, the problem created in this combination scenario is to find the shortest path in order to visit each city and stop there for the desired purpose at a time, and then return to the city from which the travel was started. The desired purpose of the journey is to minimize the total travelling distance. TSP has combinatorial complexity, and therefore heuristic or approximate solution procedures have always been carried out in practice [8-10]. The general algorithms for the solution of TSP have shown that the solution process grows exponentially as the size of the problem increases. Although TSP is a computationally hard problem, there exist many exact and heuristic algorithms in the literature that produce good solutions even if there are thousands of cities in the problem. The proper decision-making principles have been taken into account in the previous studies to handle different problems for optimal solutions [18-22]

The traveling salesman problem (TSP) is looking for a trip through a finite set of cities known as nodes that minimizes the total traveling distance. In other words, TSP is finding a Hamiltonian route of minimum cost in an edge-weighted graph. TSP is also known as an NP-hard combinatorial optimization problem that needs to be taken into account in plenty of real-world applications, directly or as a sub-optimization problem. The TSP has plenty of applications with its easy formulation, such as logistics, planning, microchip manufacturing, and many others. The modified form of TSP has been used as a sub-problem in many areas of science, for instance, DNA sequencing. The TSP has also appeared in astronomy, as astronomers are examining many places that definitely want to minimize the time spent while moving the telescope among the different places.

Traveling salesman problems are somehow similar to assignment problems [11]. In other words, an assignment problem is a specific case of the transportation problem [5-7], where the aim is to assign a number of resources to an equal number of activities to minimize total expenditures or maximize total profit of allocation. More formally, if there are n cities, and the purpose is to minimize the total path among these cities, then the traveling salesman problem (TSP) can be used, where each city can be visited only once. The main objective of a traveling salesman is to minimize the total traveling distance among n-cities [14-15]. If subscripts i and j represent two different cities, then s_{ij} can be stated as the distance from the ith city to the j^{th} city, where i = 1, 2, 3, ..., n and j = 1, 2, 3, ..., n. It should be noted that the case i=j can be illustrated as the same city. In the distance matrix, we have the following two restrictions.

1. Elements that have the same subscripts in the leading diagonal are ignored.
2. Repetition is not allowed until all the cities are visited once.

In this study, we consider two topics: the transportation problem and an assignment problem, the latter of which is put forward from the transportation problem. The assignment problem has great importance in decision making (DM) and is one of the oldest applications of LLP. Assignment problem is included in the onset applications, and a number of techniques have been applied, and many volumes have been published [1], [2, 3]. Recently, the supremely efficient point has been calculated for multi-objective optimization problems as given in [4,12]. A metaheuristic approach is presented for the solution of multi-objective assignment problems with three or more objectives in [23]. Application of the assignment problem that introduces a new route from a mega-hub to a new destination while maintaining the existing flight network and leveraging arrivals from spoke airports to ensure connectivity in flights [24].

In this paper, we propose a new approach in order to handle both mono and multi-objective assignment problems. In the first phase, the proposed methodology was carried out for the solution of a mono-objective case, while in the second phase, the mono-objective solution scenario was extended for multi-objective optimization problems.

Assignment problems have been handled through many procedures and techniques in the literature. In this scenario, the Hungarian method is one of those that has been used for the solution of an assignment problem. It should be noted that the proposed technique is carried out without a modification matrix, which reduces computational effort as compared to the Hungarian method. Moreover, in the course of the solution, the case N ≠ n does not occur, and consequently, there is no need to define a modification matrix to make N = n.

An assignment problem consisting of precisely n given free zeros is denoted by the complete cost assignment matrix of order n × n. A travelling salesman is defined as follows:

$$\sum_{i=1}^{n}\sum_{j=1}^{n} s_{ij}\, x_{ij} \qquad (1)$$

s.t

$$\begin{cases} \sum_{i=1}^{n} x_{ij} = 1, i = 1,2,3 \ldots \ldots n \\ \sum_{j=1}^{n} x_{ij} = 1, i = 1,2,3 \ldots \ldots n \\ x_{ij} = 0 \; or \; 1, i = 1,2,3 \ldots n, j = 1,2,3 \ldots n \end{cases}$$

The distance between a particular city i and a particular city j is denoted by s_{ij} , so either the conditions x_{ij}= 0 or 1 holds.

Let s_{ij} denote a distance matrix in which the distance between city i and city j is denoted by s.

$$\begin{pmatrix} s_{11} & s_{12} & s_{13} \dots . s_{1n} \\ s_{21} & s_{22} & s_{23} \dots s_{2n} \\ s_{31} & s_{32} & s_{33 \dots .} \; s_{3n} \\ . & . & . \quad . \\ . & . & . \quad . \\ s_{1n} & s_{2n} & s_{3n} \quad s_{nn} \end{pmatrix}$$

In the following, the proposed approach is presented for mono-objective optimization problems in order to solve traveling salesman problems.

2 | PHASE ONE: PROPOSED APPROACH FOR MONO-OBJECTIVE OPTIMIZATION PROBLEMS

In equation (1), sij is the distance matrix in which the distance between city i and city j is denoted by s. The proposed algorithm proceeds as follows:

Step 1: Find the sum of all the entries in each column of the given assignment cost matrix, and then find the minimum sum as S_{min}. Divide each sum of the j^{th} column of the distance matrix by S_{min}. If each sum is less than 2, that is, $k_j < 2 \; \forall j$, then proceed to step 3, proceed to step 2.

We have,

$$\begin{pmatrix} s_{11} & s_{12} & s_{13} \dots . s_{1n} \\ s_{21} & s_{22} & s_{23} \dots s_{2n} \\ s_{31} & s_{32} & s_{33 \dots .} \; s_{3n} \\ . & . & . \quad . \\ . & . & . \quad . \\ s_{1n} & s_{2n} & s_{3n} \quad s_{nn} \end{pmatrix}$$

$$S_1 = \sum_{i=1}^{n} s_{i1} \, , S_2 = \sum_{i=1}^{n} s_{i2} \, , \dots \dots . . S_n = \sum_{i=1}^{n} s_{in}$$

Divide each sum by the minimum sum S_{min} , then we have

$$R_1 = \frac{s_1}{s_{min}} , R_2 = \frac{s_2}{s_{min}} , \dots \dots \dots \dots . R_n = \frac{s_n}{s_{min}}$$

Step 2: If a number is added to or subtracted from all entries of any row or any column of the cost matrix, then the optimal assignment cost matrix is also the optimal assignment matrix for the original cost matrix [17]. In other words, a step is used to make the sum of each column entry less than 2

Step 3: Find the minimum element sij of each row of the distance matrix sij and then subtract it from the corresponding row sij to create at least one zero in each row.

$$\begin{pmatrix} s'_{11} & s'_{12} & s'_{13} \dots . s'_{1n} \\ s'_{21} & s'_{22} & s'_{23} \dots s'_{2n} \\ s'_{31} & s'_{32} & s'_{33} \dots s'_{3n} \\ . & . & . \quad . \\ . & . & . \quad . \\ s'_{1n} & s'_{2n} & s'_{3n} \dots s'_{nn} \end{pmatrix}$$

$$\min (s'_{11} \quad s'_{12} \quad s'_{13} \dots s'_{1n})$$
$$\min (s'_{21} \quad s'_{22} \quad s'_{23} \dots s'_{2n})$$
$$\min (s'_{31} \quad s'_{32} \quad s'_{33} \dots s'_{3n})$$
……………………………...
………………………………
………………………………
$$\min (s'_{1n} \quad s'_{2n} \quad s'_{3n} \dots s'_{3n})$$

Step 4: Similarly, find the minimum element sij of each column of the distance matrix sij and then subtract it from the corresponding column sij to create at least one zero in each column.

$$\begin{pmatrix} s''_{11} & s''_{12} & s''_{13} \dots . s''_{1n} \\ s''_{21} & s''_{22} & s''_{23} \dots s''_{2n} \\ s''_{31} & s''_{32} & s''_{33} \dots s''_{3n} \\ . & . & . \quad . \\ . & . & . \quad . \\ s''_{1n} & s''_{2n} & s''_{3n} \dots s''_{nn} \end{pmatrix}$$

$$\min (s''_{11} \quad s''_{12} \quad s''_{13} \dots s''_{1n})/$$
$$\min (s''_{21} \quad s''_{22} \quad s''_{23} \dots s''_{2n})/$$
$$\min (s''_{31} \quad s''_{32} \quad s''_{33} \dots s''_{3n})/$$
……………………………...
………………………………
………………………………
$$\min (s''_{1n} \quad s''_{2n} \quad s''_{3n} \dots s''_{3n})/$$

Step 5: If the number of lines N (lines through any row or column that contains at least one zero) is equal to n (distance matrix's order), then a solution exists; otherwise, solution does not exist.

$$\begin{pmatrix} s^{///}_{11} & s^{///}_{12} & s^{///}_{13} \ldots . s^{///}_{1n} \\ s^{///}_{21} & s^{///}_{22} & s^{///}_{23} \ldots s^{///}_{2n} \\ s^{///}_{31} & s^{///}_{32} & s^{///}_{33} \ldots s^{///}_{3n} \\ . & . & . \quad . \\ . & . & . \quad . \end{pmatrix}$$

3 | EXAMPLES

In order to test the validity of the proposed methodology by computational experiments, we will test the proposed algorithm by taking mono-objective assignment problems from the existing literature.

Example 1: The travelling distance of a salesman to five different cities is given in the following matrix. Minimize the total traveling distance of the salesman by using the proposed methodology.

$$\begin{array}{c c} & \begin{array}{ccccc} C_1 & C_2 & C_3 & C_4 & C_5 \end{array} \\ \begin{array}{c} C_1 \\ C_2 \\ C_3 \\ C_4 \\ C_5 \end{array} & \begin{pmatrix} - & 11 & 10 & 12 & 4 \\ 2 & - & 6 & 3 & 5 \\ 3 & 12 & - & 14 & 6 \\ 6 & 14 & 4 & - & 7 \\ 7 & 9 & 8 & 12 & - \end{pmatrix} \end{array}$$

$$\begin{array}{c c} & \begin{pmatrix} - & 11 & 10 & 12 & 4 \\ 2 & - & 6 & 3 & 5 \\ 3 & 12 & - & 14 & 6 \\ 6 & 14 & 4 & - & 7 \\ 7 & 9 & 8 & 12 & - \end{pmatrix} \\ \hline \text{sums} & \begin{array}{ccccc} 18 & 46 & 28 & 41 & 22 \end{array} \\ k_j & \begin{array}{ccccc} 1 & 2.5 & 1.5 & 2.2 & 1.2 \end{array} \end{array}$$

To make the sum of each column less than 2, subtract 3 from the second and fourth columns, respectively. Then the distance matrix becomes:

$$
\begin{array}{cc}
 & \begin{pmatrix} - & 8 & 10 & 9 & 4 \\ 2 & - & 6 & 0 & 5 \\ 3 & 9 & - & 11 & 6 \\ 6 & 11 & 4 & - & 7 \\ 7 & 6 & 8 & 9 & - \end{pmatrix} \\
\text{sum} & \begin{matrix} 18 & 34 & 28 & 29 & 22 \end{matrix} \\
k_j & \begin{matrix} 1 & 1.8 & 1.5 & 1.6 & 1.2 \end{matrix}
\end{array}
$$

The net cost matrix is as follows:

$$
\begin{pmatrix} - & 8 & 10 & 9 & 4 \\ 2 & - & 6 & 0 & 5 \\ 3 & 9 & - & 11 & 6 \\ 6 & 11 & 4 & - & 7 \\ 7 & 6 & 8 & 9 & - \end{pmatrix}
$$

From step 3, find the minimum row entries;

$$
\left(\begin{array}{ccccc|c} - & 8 & 10 & 9 & 4 & 4 \\ 2 & - & 6 & 0 & 5 & 0 \\ 3 & 9 & - & 11 & 6 & 3 \\ 6 & 11 & 4 & - & 7 & 4 \\ 7 & 6 & 8 & 9 & - & 6 \end{array}\right)
$$

As a result, we have,

$$
\begin{pmatrix} - & 4 & 6 & 5 & 0 \\ 2 & - & 6 & 0 & 5 \\ 0 & 6 & - & 8 & 3 \\ 2 & 7 & 0 & - & 3 \\ 1 & 0 & 2 & 3 & - \end{pmatrix}
$$

From step 4, the minimum column elements are given by;

$$
\begin{array}{cc}
 & \begin{pmatrix} - & 4 & 6 & 5 & 0 \\ 2 & - & 6 & 0 & 5 \\ 0 & 6 & - & 8 & 3 \\ 2 & 7 & 0 & - & 3 \\ 1 & 0 & 2 & 3 & - \end{pmatrix} \\
\text{Min} & \begin{matrix} 0 & 0 & 0 & 0 & 0 \end{matrix}
\end{array}
$$

From step 4, find the minimum column entries;

$$\begin{pmatrix} - & 4 & 6 & 5 & 0 \\ 2 & - & 6 & 0 & 5 \\ 0 & 6 & - & 8 & 3 \\ 2 & 7 & 0 & - & 3 \\ 1 & 0 & 2 & 3 & - \end{pmatrix}$$

$$\boldsymbol{N = n}$$

$$\begin{pmatrix} - & 4 & 6 & 5 & 0 \\ 2 & - & 6 & 0 & 5 \\ 0 & 6 & - & 8 & 3 \\ 2 & 7 & 0 & - & 3 \\ 1 & 0 & 2 & 3 & - \end{pmatrix}$$

Therefore, the sales man travelled from C1 to C5, C5 to C2, C2 to C4, C4 to C3 and then C3 to C1, that is $C_1\ C_5\ C_2\ C_4\ C_3\ C_1$

= 4 + 9 + 3 + 4 + 3 = 23

Example 2: A travelling salesman delivers products to five different cities. Minimize the total travelling distance using the proposed methodology.

$$\begin{array}{c} \\ C_1 \\ C_2 \\ C_3 \\ C_4 \\ C_5 \end{array} \begin{array}{c} \begin{array}{ccccc} C_1 & C_2 & C_3 & C_4 & C_5 \end{array} \\ \begin{pmatrix} - & 600 & 1000 & 1900 & 1100 \\ 600 & - & 1900 & 1900 & 1500 \\ 1000 & 1900 & - & 1700 & 1200 \\ 1900 & 1900 & 1700 & - & 190 \\ 1100 & 1500 & 1200 & 1900 & - \end{pmatrix} \end{array}$$

$$\begin{array}{cccccc} & \begin{pmatrix} - & 600 & 1000 & 1900 & 1100 \\ 600 & - & 1900 & 1900 & 1500 \\ 1000 & 1900 & - & 1700 & 1200 \\ 1900 & 1900 & 1700 & - & 1900 \\ 1100 & 1500 & 1200 & 1900 & - \end{pmatrix} \\ \hline \text{Sum} & 3600 \quad 5900 \quad 5800 \quad 7400 \quad 5700 \\ \text{kj} & 1 \quad 1.64 \quad 1.6 \quad 2.06 \quad 1.58 \end{array}$$

Now subtract 200 from the fourth column to make each sum less than 2. So, the matrix becomes

$$\begin{array}{c c} & \begin{pmatrix} - & 600 & 1000 & 1700 & 1100 \\ 600 & - & 1900 & 1700 & 1500 \\ 1000 & 1900 & - & 1500 & 1200 \\ 1900 & 1900 & 1700 & - & 1900 \\ 1100 & 1500 & 1200 & 1700 & - \end{pmatrix} \\ \hline \text{Sums} & \begin{matrix} 3600 & 5900 & 5800 & 6600 & 5700 \end{matrix} \\ \text{kj} & \begin{matrix} 1 & 1.64 & 1.6 & 1.83 & 1.5 \end{matrix} \end{array}$$

$$\begin{pmatrix} - & 600 & 1000 & 1700 & 1100 \\ 600 & - & 1900 & 1700 & 1500 \\ 1000 & 1900 & - & 1500 & 1200 \\ 1900 & 1900 & 1700 & - & 1900 \\ 1100 & 1500 & 1200 & 1700 & - \end{pmatrix}$$

$$\left(\begin{array}{ccccc|c} - & 600 & 1000 & 1700 & 1100 & 600 \\ 600 & - & 1900 & 1700 & 1500 & 600 \\ 1000 & 1900 & - & 1500 & 1200 & 1000 \\ 1900 & 1900 & 1700 & - & 1900 & 1700 \\ 1100 & 1500 & 1200 & 1700 & - & 1100 \end{array}\right)$$

$$\begin{array}{c c} & \begin{pmatrix} - & 0 & 400 & 1100 & 500 \\ 0 & - & 1300 & 1100 & 900 \\ 0 & 900 & - & 500 & 200 \\ 200 & 200 & 0 & - & 200 \\ 0 & 400 & 200 & 600 & - \end{pmatrix} \\ \hline \text{Min} & \begin{matrix} 0 & 0 & 0 & 500 & 200 \end{matrix} \end{array}$$

$$\begin{pmatrix} - & 0 & 400 & 600 & 300 \\ 0 & - & 1300 & 600 & 700 \\ 0 & 900 & - & 0 & 0 \\ 200 & 200 & 0 & - & 0 \\ 0 & 400 & 200 & 100 & - \end{pmatrix}$$

$$N=n$$

Therefore, the desired description is given by;

$$\begin{pmatrix} - & 0 & 400 & 600 & 300 \\ 0 & - & 1300 & 600 & 700 \\ 0 & 900 & - & 0 & 0 \\ 200 & 200 & 0 & - & 0 \\ 0 & 400 & 200 & 100 & - \end{pmatrix}$$

As a result, the journey of the salesman is given by,

The sales man travelled from C_1 to C_5, C_5 to C_3, C_3 to C_4, C_4 to C_2 and then C_2 to C_1, that is

$$C_1 \quad C_5 \quad C_3 \quad C_4 \quad C_2 \quad C_1$$

$$=1100 + 1200 + 1700 + 1900 + 600 = 6500$$

Example 3: The distance among five cities is given by the following matrix. Minimize the total traveling distance among these cities.

$$\begin{array}{cc} & \begin{array}{ccccc} C_1 & C_2 & C_3 & C_4 & C_5 \end{array} \\ \begin{array}{c} C_1 \\ C_2 \\ C_3 \\ C_4 \\ C_5 \end{array} & \begin{pmatrix} - & 10 & 3 & 6 & 9 \\ 5 & - & 5 & 4 & 2 \\ 4 & 9 & - & 7 & 8 \\ 7 & 1 & 3 & - & 4 \\ 3 & 2 & 6 & 5 & - \end{pmatrix} \\ \text{Sum} & \begin{array}{ccccc} 19 & 22 & 17 & 22 & 23 \end{array} \\ k_j & \begin{array}{ccccc} 1.1 & 1.2 & 1 & 1.2 & 1.3 \end{array} \end{array}$$

From step 3, the minimum row elements are given

$$\left(\begin{array}{ccccc|c} - & 10 & 3 & 6 & 9 & 3 \\ 5 & - & 5 & 4 & 2 & 2 \\ 4 & 9 & - & 7 & 8 & 4 \\ 7 & 1 & 3 & - & 4 & 1 \\ 3 & 2 & 6 & 5 & - & 2 \end{array}\right)$$

$$\begin{pmatrix} - & 7 & 0 & 3 & 6 \\ 3 & - & 3 & 2 & 0 \\ 0 & 5 & - & 3 & 4 \\ 6 & 0 & 2 & - & 3 \\ 1 & 0 & 4 & 3 & - \end{pmatrix}$$

From step 4, we have, i.e., the minimum column elements are given by;

$$\begin{pmatrix} - & 7 & 0 & 3 & 6 \\ 3 & - & 3 & 2 & 0 \\ 0 & 5 & - & 3 & 4 \\ 6 & 0 & 2 & - & 3 \\ 1 & 0 & 4 & 3 & - \end{pmatrix}$$
$$\min \quad 0 \quad 0 \quad 0 \quad 2 \quad 0$$

$$\begin{pmatrix} - & 7 & 0 & 1 & 6 \\ 3 & - & 3 & 0 & 0 \\ 0 & 5 & - & 1 & 4 \\ 6 & 0 & 2 & - & 3 \\ 1 & 0 & 4 & 1 & - \end{pmatrix}$$

$$\begin{pmatrix} - & 7 & 0 & 1 & 6 \\ 3 & - & 3 & 0 & 0 \\ 0 & 5 & - & 1 & 4 \\ 6 & 0 & 2 & - & 3 \\ 1 & 0 & 4 & 1 & - \end{pmatrix}$$

$$\boldsymbol{N \neq n}$$

Here $\boldsymbol{N \neq n}$, where N represents the number of lines in the distance matrix that contains at least one zero, and n represents the order of the matrix. In this case, we will find the modification matrix. For this, choose the smallest element 1 that does not lie on the lines N and subtract it from the remaining elements of the matrix until the number of lines N becomes equal to n, as given below;

$$\begin{pmatrix} - & 7 & 0 & 0 & 5 \\ 3 & - & 3 & 0 & 0 \\ 0 & 5 & - & 0 & 3 \\ 6 & 0 & 2 & - & 2 \\ 1 & 0 & 4 & 0 & - \end{pmatrix}$$

$$\boldsymbol{N = n}$$

In order to calculate distances among these cities, we will use priority rules as follows.

$$\begin{pmatrix} - & 7 & 0 & 0 & 5 \\ 3 & - & 3 & 0 & 0 \\ 0 & 5 & - & 0 & 3 \\ 6 & 0 & 2 & - & 2 \\ 1 & 0 & 4 & 0 & - \end{pmatrix}$$

Here the salesman traveled from C_1 to C_3, C_3 to C_4, C_4 to C_2, C_2 to C_5 and then C_5 to C_1, that is

$$C_1 \; C_3 \; C_4 \; C_2 \; C_5 \; C_1$$

$$= 3 + 7 + 1 + 2 + 3 = 16$$

4 | PHASE TWO: PROPOSED APPROACH FOR MULTI-OBJECTIVE OPTIMIZATION PROBLEMS

In the second phase of this paper, the proposed methodology is extended for multi-objective optimization problems where the desired trade-off solutions are known as Pareto optimal or Pareto efficient solutions. To apply the proposed methodology for multi-objective optimization problems, we first introduce some basic definitions and solution terminology in such a scenario.

Pareto efficiency: The concept of Pareto optimality was introduced in the economic equilibrium and welfare theories at the beginning of the 19th century. The main idea of this concept is that a society is enjoying maximum ophelimity when no one can be made better off without making someone else worse off. This topic is related to an Italian economist, Vilfredo Pareto (1848-1923). He introduced this idea during the investigations of monetary productivity and pay dispersion. This idea has been connected in scholarly fields such as financial matters, buildings, and the existence sciences, specially in the field of multi-objective optimization.

Pareto frontier: The Pareto frontier is the arrangement of all Pareto efficient allocations or solution points that ordinarily appear. The combined trade-off is known as the Pareto front, or Pareto set [12].

Pareto improvement: A Pareto improvement is a change to an alternative to make one individual better, and the other worse. An assignment is characterized as" Pareto productive" or "Pareto optimal" when no further Pareto improvement can be put forth; this leads the problem to Pareto optimality.

Pareto Optimality: Pareto optimality is a change to an alternative that makes one individual better off without making some other individual worse off. An assignment is characterized as "Pareto improvement " or "Pareto ideal" when no further Pareto upgrades can be put forth; with that, we can accept the solution, in other words, we have achieved Pareto optimality.

A weak Pareto optimality is an assignment for which there are no conceivable elective distributions whose acknowledgment would make each gain [13]. Thus, an elective allotment is viewed as a Pareto enhancement if and only if the elective portion is entirely favored by all people.

Constrained Pareto optimality: The state of constrained Pareto optimality is a weaker rendition of the standard state of Pareto optimality used in financial aspects, which apparently represents the way that a potential organizer will most likely be unable to enhance a decentralized market result, regardless of whether that result is not good. This will happen in the event that it is restricted by identical educational or institutional requirements from individual agents [15].

Pareto set: The Pareto set is an arrangement or allotment of all those Pareto efficient points that constitute Pareto frontiers, especially valuable in designing. By yielding the majority of the possibly ideal arrangements, a planner can make centered tradeoffs within this compelled set of parameters, instead of expecting to think about the whole scope of parameters.

Pareto front: A lot of non-dominated arrangements are picked as optimal if no further improvement is possible, which constitutes the Pareto front. All Pareto optimal solutions x* constitute the Pareto front, and no further improvement can be made in this set.

Generally, a multi-objective assignment problem can be formulated as follows:

$$min\, z_k(x) = \sum_{i=1}^{n}\sum_{j=1}^{n} c^k{}_{ij}\, x_{ij}, k = 1,2,3 \ldots .\, p$$

s.t

$$\begin{cases} \sum_{i=1}^{n} x_{ij} = 1, i = 1,2,3 \ldots \ldots n \\ \sum_{j=1}^{n} x_{ij} = 1, i = 1,2,3 \ldots \ldots n \\ x_{ij} \varepsilon \{0,1\}, (i,j) = 1,2,3 \ldots n \end{cases}$$

Where all coefficient c_{ij}^k in the objective function are positive integers. If we don't repeat S, then the index matrix to the goal function of the kth is shown by ck and the set of Z(x) = (z1(x), z2(x), ... zp(x)) where x = (x11, x12...xnn)ε S represent the feasible region Y in the target space.

Definition 1: A point x∗ ∈ S is said to be an efficient solution when there does not exist x ∈ S such that ∀ i ∈ {1, 2, ...n}; zi(x) ≤ zi(x∗) and ∃ j ∈ {1, 2, ...n}; zi(x) < zj(x∗). Then Z *(x) is said to be a non- dominated solution point.

Definition 2: Assume xi∗ ∈ S for each i ∈ {1, 2,..., p} is an optimum point of zi(x) in the space S, then the vector (z1(x1∗), z2(x2∗),..., zp(xp∗) in the objective space is known as an ideal point.

Definition 3: An efficient point $x* \in S$ is said to be super-efficient when $Z*(x)$ has the minimum distance from an ideal point; then $Z(x*)$ is said to be the best non-dominated point.

In the following, we validate the proposed methodology for multi-objective problems with a numerical example. The ideal solution of each objective can be determined separately by the proposed methodology, as discussed in the phase, and then the Pareto-efficient point of the multi-objective problem of the competing objectives is determined.

Example 4: Consider the three-objective assignment problem, which is given by three-cost matrices,

$$C^1 = \begin{pmatrix} 2 & 5 & 4 & 7 \\ 3 & 3 & 5 & 7 \\ 3 & 8 & 4 & 2 \\ 6 & 5 & 2 & 5 \end{pmatrix}$$

$$C^2 = \begin{pmatrix} 3 & 3 & 6 & 2 \\ 5 & 3 & 7 & 2 \\ 5 & 2 & 7 & 4 \\ 4 & 6 & 3 & 5 \end{pmatrix}$$

$$C^3 = \begin{pmatrix} 4 & 2 & 5 & 3 \\ 5 & 3 & 4 & 3 \\ 4 & 3 & 5 & 2 \\ 6 & 4 & 7 & 3 \end{pmatrix}$$

And,

$$C = C^1 + C^2 + C^3$$

$$\begin{pmatrix} 9 & 10 & 15 & 12 \\ 13 & 9 & 16 & 13 \\ 12 & 13 & 16 & 8 \\ 16 & 18 & 14 & 10 \end{pmatrix}$$

This multi-objective problem is solved in [13,16] by a two-phase method by using many algorithms in a long computational time, as well as many iterations. In the following, we use the proposed assignment method for multi-objective optimization problems as stated in phase one. The proposed methodology is carried out to solve each objective of the multi-objective problem separately, and then solved as a single objective assignment cost matrix C. The solution of each cost matrix is obtained as f

$$x^* = \begin{cases} x_{11} = x_{22} = x_{34} = x_{43} = 1 \\ x_{ij} = 0, \quad otherwise \end{cases}$$

and the optimum point of the objective function is Z(x∗) = (9, 13, 16). As demonstrated in the scenario of multi-objective optimization, x∗ is the aggregate solution, and Z(x∗) is the optimal super-efficient solution of the three-objective assignment problem.

In the following, we just explain the individual solution of each objective by the proposed algorithm.

The first objective is minimized as follows.

$$\begin{array}{c|cccc} C^1 = & \multicolumn{4}{c}{\begin{pmatrix} 2 & 5 & 4 & 7 \\ 3 & 3 & 5 & 7 \\ 3 & 8 & 4 & 2 \\ 6 & 5 & 2 & 5 \end{pmatrix}} \\ \hline \text{Sum} & 14 & 21 & 15 & 21 \\ k_j & 1 & 1.5 & 1.07 & 1.5 \end{array}$$

From step 3, the minimum row elements are given by;

$$\left(\begin{array}{cccc|c} 2 & 5 & 4 & 7 & 2 \\ 3 & 3 & 5 & 7 & 3 \\ 3 & 8 & 4 & 2 & 2 \\ 6 & 5 & 2 & 5 & 2 \end{array}\right)$$

$$\begin{pmatrix} 0 & 3 & 2 & 5 \\ 0 & 0 & 2 & 5 \\ 1 & 6 & 2 & 0 \\ 2 & 3 & 0 & 3 \end{pmatrix}$$

From step 4, the minimum column elements are given by;

$$
\begin{array}{r c}
 & \begin{pmatrix} 0 & 3 & 2 & 5 \\ 0 & 0 & 2 & 5 \\ 1 & 6 & 2 & 0 \\ 2 & 3 & 0 & 3 \end{pmatrix} \\
\hline
\text{min} & \begin{matrix} 0 & 0 & 0 & 0 \end{matrix}
\end{array}
$$

$$
\begin{pmatrix} 0 & 3 & 2 & 5 \\ 0 & 0 & 2 & 5 \\ 1 & 6 & 2 & 0 \\ 2 & 3 & 0 & 3 \end{pmatrix}
$$

$$\boldsymbol{N = n}$$

Where N represents the number of lines consisting of at least one zero either in a row or in a column, and n represents the order of the distance matrix. The desired solution is given by:

$$
\begin{pmatrix} 0 & 3 & 2 & 5 \\ 0 & 0 & 2 & 5 \\ 1 & 6 & 2 & 0 \\ 2 & 3 & 0 & 3 \end{pmatrix}
$$

The second objective is minimized as follows

$$
\begin{array}{r c}
 & \begin{pmatrix} 3 & 3 & 6 & 2 \\ 5 & 3 & 7 & 2 \\ 5 & 2 & 7 & 4 \\ 4 & 6 & 3 & 5 \end{pmatrix} \\
\hline
\text{sum} & \begin{matrix} 17 & 14 & 23 & 13 \end{matrix} \\
\text{k} & \begin{matrix} 1.3 & 1.07 & 1.7 & 1 \end{matrix}
\end{array}
$$

From step 3, the minimum row elements are given by;

$$
\left(\begin{array}{cccc|c} 3 & 3 & 6 & 2 & 2 \\ 5 & 3 & 7 & 2 & 2 \\ 5 & 2 & 7 & 4 & 2 \\ 4 & 6 & 3 & 5 & 3 \end{array}\right)
$$

$$\begin{pmatrix} 1 & 1 & 4 & 0 \\ 3 & 1 & 5 & 0 \\ 3 & 0 & 5 & 2 \\ 1 & 3 & 0 & 2 \end{pmatrix}$$

From step 4, the minimum column elements are given by;

$$\begin{array}{c c} & \begin{pmatrix} 1 & 1 & 4 & 0 \\ 3 & 1 & 5 & 0 \\ 3 & 0 & 5 & 2 \\ 1 & 3 & 0 & 2 \end{pmatrix} \\ \text{Min} & \begin{matrix} 1 & 0 & 0 & 0 \end{matrix} \end{array}$$

$$\begin{pmatrix} 0 & 1 & 4 & 0 \\ 2 & 1 & 5 & 0 \\ 2 & 0 & 5 & 2 \\ 0 & 3 & 0 & 2 \end{pmatrix}$$

$$\boldsymbol{N = n}$$

Therefore, the desired solution by the proposed algorithm is given by;

$$\begin{pmatrix} 0 & 1 & 4 & 0 \\ 2 & 1 & 5 & 0 \\ 2 & 0 & 5 & 2 \\ 0 & 3 & 0 & 2 \end{pmatrix}$$

The third objective is minimized as

$$\begin{pmatrix} 4 & 2 & 5 & 3 \\ 5 & 3 & 4 & 3 \\ 4 & 3 & 5 & 2 \\ 6 & 4 & 7 & 3 \end{pmatrix}$$

$$\begin{array}{c c} & \begin{pmatrix} 4 & 2 & 5 & 3 \\ 5 & 3 & 4 & 3 \\ 4 & 3 & 5 & 2 \\ 6 & 4 & 7 & 3 \end{pmatrix} \\ \text{Sums} & \begin{matrix} 19 & 12 & 21 & 11 \end{matrix} \\ k_j & \begin{matrix} 1.7 & 1 & 1.9 & 1.1 \end{matrix} \end{array}$$

And the net cost matrix is as follows:

$$6\begin{pmatrix} 4 & 2 & 5 & 3 \\ 5 & 3 & 4 & 3 \\ 4 & 3 & 5 & 2 \\ 4 & 7 & 3 & \end{pmatrix}$$

From step 3, the minimum row elements are given

$$\left(\begin{array}{cccc|c} 4 & 2 & 5 & 3 & 2 \\ 5 & 3 & 4 & 3 & 3 \\ 4 & 3 & 5 & 2 & 2 \\ 6 & 4 & 7 & 3 & 3 \end{array}\right)$$

$$\begin{pmatrix} 2 & 0 & 3 & 1 \\ 2 & 0 & 1 & 0 \\ 2 & 1 & 3 & 0 \\ 3 & 1 & 4 & 0 \end{pmatrix}$$

From step 4, the minimum column elements are given by;

$$\begin{array}{cc} & \begin{pmatrix} 2 & 0 & 3 & 1 \\ 2 & 0 & 1 & 0 \\ 2 & 1 & 3 & 0 \\ 3 & 1 & 4 & 0 \end{pmatrix} \\ \hline \text{min} & \begin{matrix} 2 & 0 & 0 & 0 \end{matrix} \end{array}$$

$$\begin{pmatrix} 0 & 0 & 3 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 3 & 0 \\ 1 & 1 & 4 & 0 \end{pmatrix}$$

$$\boldsymbol{N = n}$$

Where N represents the number of lines consisting of at least one zero either in a row or in a column, and n represents the order of the distance matrix. The desired solution is given by:

$$\begin{pmatrix} 0 & 0 & 3 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 3 & 0 \\ 1 & 1 & 4 & 0 \end{pmatrix}$$

- x^1 with $x_{11} = x_{22} = x_{34} = x_{43} = 0$, and the objective value is $y^1 = (9, 13, 16)$, that is the minimum solution vector of the first objective.
- x^2 with $x_{11} = x_{24} = x_{32} = x_{43} = 0$, and the objective value is $y^2 = (19, 10, 17)$, that is the minimum solution vector of the second objective.
- x^3 with $x_{12} = x_{23} = x_{31} = x_{44} = 0$, and the objective value is $y^3 = (18, 20, 13)$, that is the minimum solution vector of the third objective.

From this problem, we can see that the best non-dominated solution is determined by solving a single- objective assignment problem in less number of iterations than [16]. The resultant non-dominated solution is evaluated through the proposed method, which produces better results. The ideal objective point of the multi-objective assignment problem, having three single-objective assignment problems, is calculated, which yields I = (9, 11, 13). In the following problem taken from [16], all the non-dominated points are calculated for the efficient point which have minimum distance from the ideal point. This problem takes a lot of computational time, which yields the following non-dominated solutions.

$$y^1 = (9, 13, 16)\ s^1 = 5,$$
$$y^2 = (19, 11, 17) s^2 = 14,$$
$$y^3 = (18, 20, 13) s^3 = 14,$$
$$y^4 = (20, 17, 14) s^4 = 18,$$
$$y^5 = (14, 20, 14) s^5 = 15,$$
$$y^6 = (18, 18, 14) s^6 = 17,$$
$$y^7 = (14, 18, 15) s^7 = 18$$

Where s^j represents the distance of the super-efficient solution of jth from the ideal objective vector with the standard L_1 norm, and the best non-dominated point is $y^1 = Z(x*)$. Similar arrangements can be given for the remaining points.

5 | CONCLUSION

In the first phase of this paper, we have presented a new method for the solution of mono-objective assignment problems. This new approach is somewhat similar to the Hungarian method; however, the case N ≠ n was not happen. Therefore, we need not define a modification that differentiates the presented technique from the Hungarian algorithm in this sense. Moreover, the proposed algorithm minimizes the iterative procedure as compared to the Hungarian method, where we need to define a modification matrix in some cases. In the second phase of this paper, the proposed algorithm, which was carried out for mono-objective assignment problems, is equally applicable for multi-objective assignment problems. The competing objectives in a multi-objective assignment problem have been transferred to a single cost matrix, and then solved by the proposed methodology that was used in the first phase. The numerical outcomes in both cases produced promising results.

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